Ms. McCleary’s “Problems to Ponder”  
[non-calculator problems]  

How do problems #1 and #2 differ from each other?

1. The polynomial function $f$ has selected values for its second derivative $f''$ given in the table below. Based on the table, which of the following statements must be true?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f''$</td>
<td>5</td>
<td>0</td>
<td>-7</td>
<td>4</td>
</tr>
</tbody>
</table>

(A) $f$ is increasing on the interval $(0, 2)$
(B) $f$ is decreasing on the interval $(0, 2)$
(C) $f$ has a local maximum at $x = 1$
(D) The graph of $f$ has a point of inflection at $x = 1$
(E) The graph of $f$ changes concavity in the interval $(0, 2)$

2. The function $f$ has selected values for its second derivative $f''$ given in the table below. $f''$ is a strictly decreasing function for the interval $[0, 3]$. Based on the table, which of the following statements must be true?

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f''$</td>
<td>5</td>
<td>0</td>
<td>-3</td>
<td>-4</td>
</tr>
</tbody>
</table>

(A) $f$ is increasing on the interval $(0, 2)$
(B) $f$ is decreasing on the interval $(0, 2)$
(C) $f$ has a local maximum at $x = 1$
(D) The graph of $f$ has a point of inflection at $x = 1$
(E) The graph of $f$ changes concavity in the interval $(0, 2)$

How do problems #3 and #4 differ from each other?

3. The function $f$ is continuous for $-2 \leq x \leq 2$ with selected values given in the table below. Based on the table, which of the following statements must be true?

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>5</td>
<td>-5</td>
<td>0</td>
</tr>
</tbody>
</table>

(A) There is some $c, -2 < c < 2$ such that $f'(c) = 0$
(B) There is some $c, -2 < c < 2$ such that $f(c) = 10$
(C) There is some $c, -2 < c < 2$ such that $f(c) = -10$
(D) There is some $c, -1 < c < 0$ such that $f(c) = 0$
(E) There is some $c, -2 < c < 2$ such that $f''(c) = 0$
4. The function $f$ is differentiable for $-2 \leq x \leq 2$ with selected values given in the table below. Based on the table, which of the following statements must be true?

(A) There is some $c, -2 < c < 2$ such that $f'(c) = 0$
(B) There is some $c, -2 < c < 2$ such that $f(c) = 10$
(C) There is some $c, -2 < c < 2$ such that $f(c) = -10$
(D) There is some $c, -1 < c < 0$ such that $f'(c) = 0$
(E) There is some $c, -1 < c < 0$ such that $f''(c) = 0$

How can you easily get “tricked” by problem #5?
5. Let $f$ be a twice-differentiable function whose second derivative, $f''$ is defined by

$$f''(x) = x(x + 3)(x - 3)^2.$$ Where does $f$ have point(s) of inflection?

(A) At $x = 0$ only
(B) At $x = -3$ only
(C) At $x = 0$ and $x = -3$ only
(D) At $x = 0$, $x = -3$, and $x = 3$
(E) At $x = 0$ and $x = 3$ only

Don’t get “horizontal” and “vertical” mixed up! [Likewise, $-\infty$ and $\infty$]
6. Let $f$ be the function defined by $f(x) = \frac{x^2 - 4}{x^2 + x - 2}$. Which of the following statements is/are true about $f$?

I. $\lim_{x \to -2} f(x)$ does not exist
II. $y = 1$ is a horizontal asymptote of $f$
III. $x = 1$ is a vertical asymptote of $f$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III
7. What are all of the horizontal asymptotes of the graph of \( y = \frac{7-3^x}{14+3^x} \)?

(A) \( y = 0 \) only
(B) \( y = \frac{1}{2} \) only
(C) \( y = -1 \) only
(D) \( y = 0 \) and \( y = -1 \)
(E) \( y = 0, \ y = \frac{1}{2}, \) and \( y = -1 \)

**What do you mean by “average”?**

8. Let \( f \) be the function defined by \( f(x) = \frac{1}{x^2} \). What is the average rate of change for the closed interval \([1, 2]\)?

(A) 0.75
(B) \(-0.75\)
(C) 0.50
(D) -0.50
(E) 1

9. What is the average value of \( y = \frac{1}{x^2} \) on the closed interval \([1, 2]\)?

(A) 0.75
(B) \(-0.75\)
(C) 0.50
(D) -0.50
(E) 1

10. Let \( f \) be a twice-differentiable function on the closed interval \([3, 7]\) with \( f(3) = f(7) \). Which of the following statements must be true about \( f \)?

(A) \( f \) is concave down on \([3, 7]\)
(B) There is a \( c, \ 3 < c < 7 \), such that \( f'(c) = 0 \)
(C) There is a \( c, \ 3 < c < 7 \), such that \( f''(c) = 0 \)
(D) There is a \( c, \ 3 < c < 7 \), such that \( f''(c) = 0 \)
(E) \( f'(3) > f'(7) \)
Those motion problems will never go away!

A bug is crawling along a straight wire. The velocity, \( v(t) \), of the bug at time \( t \), \( 0 \leq t \leq 11 \), is given in the graph above.

11. According to the graph, at what time \( t \) does the bug change direction?

(A) 2  
(B) 5  
(C) 6  
(D) 8  
(E) 10

12. According to the graph, at what time \( t \) is the speed of the bug greatest?

(A) 2  
(B) 5  
(C) 6  
(D) 8  
(E) 10