24. From Example 3(a), we have $A = \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} e^{-2i/n}$. Using a CAS, $\sum_{i=1}^{n} e^{-2i/n} = \frac{e^{-2}(e^2-1)}{e^{2/n}-1}$ and

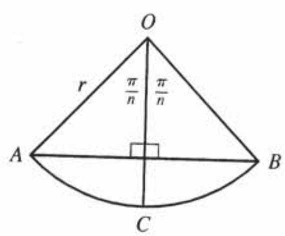
$\lim_{n\to\infty} \frac{2}{n}\cdot\frac{e^{-2}(e^2-1)}{e^{2/n}-1} = e^{-2}(e^2-1) \approx 0.8647$, whereas the estimate from Example 3(b) using M_{10} was 0.8632.

25. $y = f(x) = \cos x$. $\Delta x = \frac{b-0}{n} = \frac{b}{n}$ and $x_i = 0 + i\,\Delta x = \frac{bi}{n}$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\cos\left(\frac{bi}{n}\right)\cdot\frac{b}{n} \overset{\text{CAS}}{=} \lim_{n\to\infty}\left[\frac{b\sin\left(b\left(\frac{1}{2n}+1\right)\right)}{2n\sin\left(\frac{b}{2n}\right)} - \frac{b}{2n}\right] \overset{\text{CAS}}{=} \sin b$$

If $b = \frac{\pi}{2}$, then $A = \sin\frac{\pi}{2} = 1$.

26. (a)



The diagram shows one of the n congruent triangles, $\triangle AOB$, with central angle $2\pi/n$. O is the center of the circle and AB is one of the sides of the polygon. Radius OC is drawn so as to bisect $\angle AOB$. It follows that OC intersects AB at right angles and bisects AB. Thus, $\triangle AOB$ is divided into two right triangles with legs of length $\frac{1}{2}(AB) = r\sin(\pi/n)$ and $r\cos(\pi/n)$.

$\triangle AOB$ has area $2\cdot\frac{1}{2}[r\sin(\pi/n)][r\cos(\pi/n)] = r^2\sin(\pi/n)\cos(\pi/n) = \frac{1}{2}r^2\sin(2\pi/n)$, so $A_n = n\cdot\text{area}(\triangle AOB) = \frac{1}{2}nr^2\sin(2\pi/n)$.

(b) To use Equation 3.4.2, $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$, we need to have the same expression in the denominator as we have in the argument of the sine function—in this case, $2\pi/n$.

$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\frac{1}{2}nr^2\sin(2\pi/n) = \lim_{n\to\infty}\frac{1}{2}nr^2\frac{\sin(2\pi/n)}{2\pi/n}\cdot\frac{2\pi}{n} = \lim_{n\to\infty}\frac{\sin(2\pi/n)}{2\pi/n}\pi r^2$. Let $\theta = \frac{2\pi}{n}$.

Then as $n\to\infty$, $\theta\to 0$, so $\lim_{n\to\infty}\frac{\sin(2\pi/n)}{2\pi/n}\pi r^2 = \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\pi r^2 = (1)\,\pi r^2 = \pi r^2$.

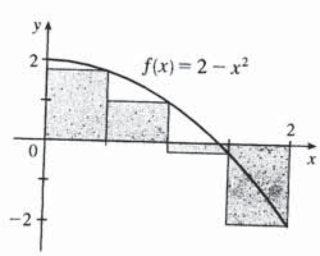
5.2 The Definite Integral

1. $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x$ $\quad[x_i^* = x_i$ is a right endpoint and $\Delta x = 0.5]$

$= 0.5\,[f(0.5) + f(1) + f(1.5) + f(2)]$ $\quad[f(x) = 2 - x^2]$

$= 0.5\,[1.75 + 1 + (-0.25) + (-2)]$

$= 0.5(0.5) = 0.25$



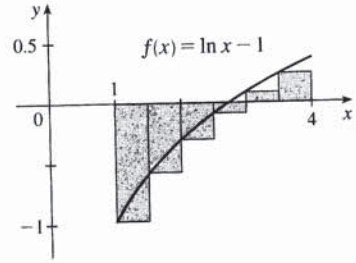
The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

2. $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \quad [x_i^* = x_{i-1}$ is a left endpoint and $\Delta x = 0.5]$

$= 0.5\,[f(1) + f(1.5) + f(2)$
$\quad + f(2.5) + f(3) + f(3.5)] \quad [f(x) = \ln x - 1]$

$\approx 0.5(-1 - 0.5945349 - 0.3068528$
$\quad - 0.0837093 + 0.0986123 + 0.2527630)$

$= 0.5(-1.6337217) \approx -0.816861$

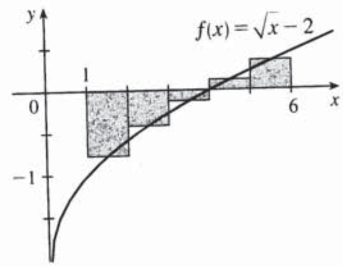


The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the four rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

3. $M_5 = \sum_{i=1}^{5} f(\overline{x}_i)\,\Delta x \quad [x_i^* = \overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$ is a midpoint and $\Delta x = 1]$

$= 1\,[f(1.5) + f(2.5) + f(3.5)$
$\quad + f(4.5) + f(5.5)] \quad [f(x) = \sqrt{x} - 2]$

≈ -0.856759

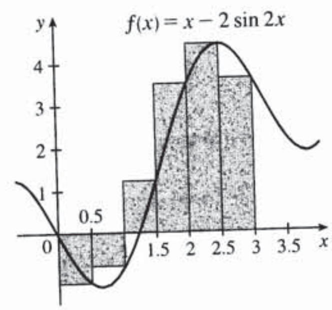


The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis.

4. (a) $R_6 = \sum_{i=1}^{6} f(x_i)\,\Delta x \quad [x_i^* = x_i$ is a right endpoint and $\Delta x = 0.5]$

$= 0.5\,[f(0.5) + f(1) + f(1.5) + f(2)$
$\quad + f(2.5) + f(3)] \quad [f(x) = x - 2\sin 2x]$

≈ 5.353254

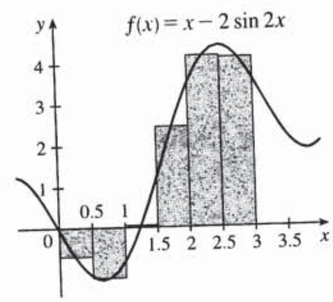


The Riemann sum represents the sum of the areas of the four rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis.

(b) $M_6 = \sum_{i=1}^{6} f(\overline{x}_i)\,\Delta x \quad [x_i^* = \overline{x}_i$ is a midpoint and $\Delta x = 0.5]$

$= 0.5[f(0.25) + f(0.75) + f(1.25) + f(1.75)$
$\quad + f(2.25) + f(2.75)] \quad [f(x) = x - 2\sin 2x]$

≈ 4.458461



The Riemann sum represents the sum of the areas of the four rectangles above the x-axis minus the sum of the areas of the two rectangles below the x-axis.

5. $\Delta x = (b - a)/n = (8 - 0)/4 = 8/4 = 2.$

(a) Using the right endpoints to approximate $\int_0^8 f(x)\,dx$, we have

$$\sum_{i=1}^{4} f(x_i)\,\Delta x = 2[f(2) + f(4) + f(6) + f(8)] \approx 2[1 + 2 + (-2) + 1] = 4.$$

(b) Using the left endpoints to approximate $\int_0^8 f(x)\,dx$, we have

$$\sum_{i=1}^{4} f(x_{i-1})\,\Delta x = 2[f(0) + f(2) + f(4) + f(6)] \approx 2[2 + 1 + 2 + (-2)] = 6.$$

(c) Using the midpoint of each subinterval to approximate $\int_0^8 f(x)\,dx$, we have

$$\sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 2[f(1) + f(3) + f(5) + f(7)] \approx 2[3 + 2 + 1 + (-1)] = 10.$$

6. (a) Using the right endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\begin{aligned}\sum_{i=1}^{6} g(x_i)\,\Delta x &= 1[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)]\\ &\approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5\end{aligned}$$

(b) Using the left endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\begin{aligned}\sum_{i=1}^{6} g(x_{i-1})\,\Delta x &= 1[g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)]\\ &\approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5 = -1\end{aligned}$$

(c) Using the midpoint of each subinterval to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\begin{aligned}\sum_{i=1}^{6} g(\overline{x}_i)\,\Delta x &= 1[g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5)]\\ &\approx 1.5 + 0 - 1 - 1.75 - 1 + 0.5 = -1.75\end{aligned}$$

7. Since f is increasing, $L_5 \le \int_0^{25} f(x)\,dx \le R_5$.

$$\begin{aligned}\text{Lower estimate} = L_5 = \sum_{i=1}^{5} f(x_{i-1})\,\Delta x &= 5[f(0) + f(5) + f(10) + f(15) + f(20)]\\ &= 5(-42 - 37 - 25 - 6 + 15) = 5(-95) = -475\end{aligned}$$

$$\begin{aligned}\text{Upper estimate} = R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x &= 5[f(5) + f(10) + f(15) + f(20) + f(25)]\\ &= 5(-37 - 25 - 6 + 15 + 36) = 5(-17) = -85\end{aligned}$$

8. (a) Using the right endpoints to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_i)\,\Delta x = 2[f(2) + f(4) + f(6)] = 2(8.3 + 2.3 - 10.5) = 0.2$$

(b) Using the left endpoints to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_{i-1})\,\Delta x = 2[f(0) + f(2) + f(4)] = 2(9.3 + 8.3 + 2.3) = 39.8$$

(c) Using the midpoint of each interval to approximate $\int_0^6 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(\overline{x}_i)\,\Delta x = 2[f(1) + f(3) + f(5)] = 2(9.0 + 6.5 - 7.6) = 15.8.$$