

6. Note that $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ and $x_i = 0 + i \Delta x = \frac{2i}{n}$.

$$\begin{aligned}\int_0^2 (2-x^2) dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 - \frac{4i^2}{n^2}\right) \left(\frac{2}{n}\right) \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[\sum_{i=1}^n 2 - \frac{4}{n^2} \sum_{i=1}^n i^2 \right] = \lim_{n \rightarrow \infty} \frac{2}{n} \left(2n - \frac{4}{n^2} \sum_{i=1}^n i^2 \right) \\ &= \lim_{n \rightarrow \infty} \left[4 - \frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right] = \lim_{n \rightarrow \infty} \left(4 - \frac{4}{3} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n} \right) \\ &= \lim_{n \rightarrow \infty} \left[4 - \frac{4}{3} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right) \right] = 4 - \frac{4}{3} \cdot 1 \cdot 2 = \frac{4}{3}\end{aligned}$$

7. $\int_0^5 (1+2x^3) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$ [$\Delta x = 5/n$ and $x_i = 5i/n$]

$$\begin{aligned}&= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + 2 \cdot \frac{125i^3}{n^3}\right) \left(\frac{5}{n}\right) = \lim_{n \rightarrow \infty} \frac{5}{n} \left[\sum_{i=1}^n 1 + \frac{250}{n^3} \sum_{i=1}^n i^3 \right] \\ &= \lim_{n \rightarrow \infty} \frac{5}{n} \left(1 \cdot n + \frac{250}{n^3} \sum_{i=1}^n i^3 \right) = \lim_{n \rightarrow \infty} \left[5 + \frac{1250}{n^4} \cdot \frac{n^2(n+1)^2}{4} \right] \\ &= \lim_{n \rightarrow \infty} \left[5 + 312.5 \cdot \frac{(n+1)^2}{n^2} \right] = \lim_{n \rightarrow \infty} \left[5 + 312.5 \left(1 + \frac{1}{n}\right)^2 \right] \\ &= 5 + 312.5 = 317.5\end{aligned}$$