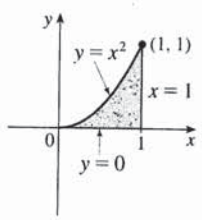
$x = 0$ or $x = \pm\sqrt{\dfrac{1}{m} - 1}$. Note that if $m = 1$, this has only the solution $x = 0$, and no region is determined. But if $1/m - 1 > 0 \Leftrightarrow 1/m > 1 \Leftrightarrow 0 < m < 1$, then there are two solutions. [Another way of seeing this is to observe that the slope of the tangent to $y = x/(x^2+1)$ at the origin is $y' = 1$ and therefore we must have $0 < m < 1$.] Note that we cannot just integrate between the positive and negative roots, since the curve and the line cross at the origin. Since mx and $x/(x^2+1)$ are both odd functions, the total area is twice the area between the curves on the interval $\left[0, \sqrt{1/m - 1}\right]$. So the total area enclosed is

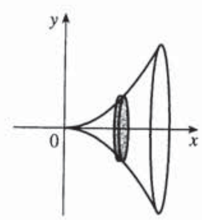
$$\begin{aligned} 2\int_0^{\sqrt{1/m-1}} \left[\frac{x}{x^2+1} - mx\right] dx &= 2\left[\tfrac{1}{2}\ln(x^2+1) - \tfrac{1}{2}mx^2\right]_0^{\sqrt{1/m-1}} \\ &= [\ln(1/m - 1 + 1) - m(1/m - 1)] - (\ln 1 - 0) \\ &= \ln(1/m) - 1 + m = m - \ln m - 1 \end{aligned}$$

6.2 Volumes

1. A cross-section is circular with radius x^2, so its area is $A(x) = \pi(x^2)^2$.

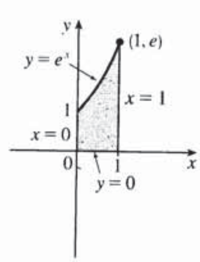
$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(x^2)^2\,dx = \pi\int_0^1 x^4 dx = \pi\left[\tfrac{1}{5}x^5\right]_0^1 = \tfrac{\pi}{5}$$

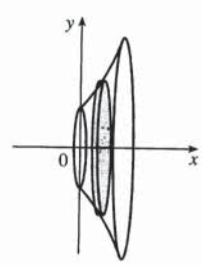




2. A cross-section is a disk with radius e^x, so its area is $A(x) = \pi(e^x)^2$.

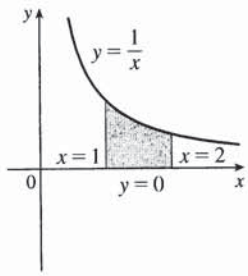
$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(e^x)^2 dx = \pi\int_0^1 e^{2x}\,dx = \tfrac{1}{2}\pi\left[e^{2x}\right]_0^1 = \tfrac{\pi}{2}(e^2 - 1)$$

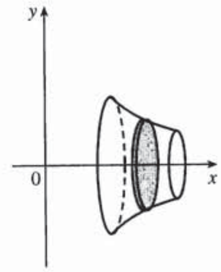




3. A cross-section is a disk with radius $1/x$, so its area is $A(x) = \pi(1/x)^2$.

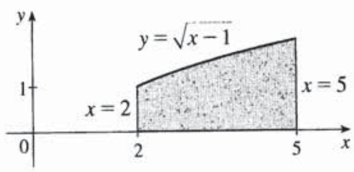
$$V = \int_1^2 A(x)\,dx = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^2 \frac{1}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^2 = \pi\left[-\tfrac{1}{2} - (-1)\right] = \tfrac{\pi}{2}$$

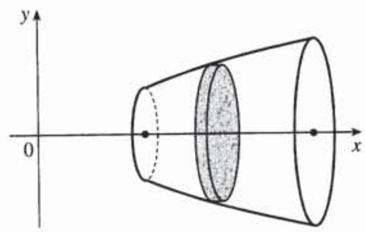




4. A cross-section is circular with radius $\sqrt{x-1}$, so its area is $A(x) = \pi\left(\sqrt{x-1}\right)^2 = \pi(x-1)$.

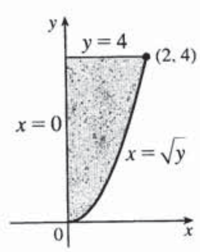
$$V = \int_2^5 A(x)\,dx = \int_2^5 \pi(x-1)\,dx = \pi\left[\tfrac{1}{2}x^2 - x\right]_2^5 = \pi\left(\tfrac{25}{2} - 5 - \tfrac{4}{2} + 2\right) = \tfrac{15}{2}\pi$$

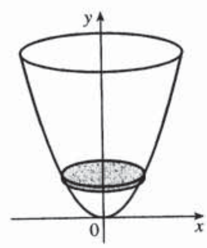




5. A cross-section is a disk with radius $\sqrt{y}$, so its area is $A(y) = \pi\left(\sqrt{y}\right)^2$.

$$V = \int_0^4 A(y)\,dy = \int_0^4 \pi\left(\sqrt{y}\right)^2 dy = \pi\int_0^4 y\,dy = \pi\left[\tfrac{1}{2}y^2\right]_0^4 = 8\pi$$

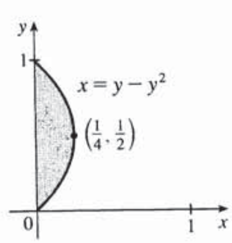


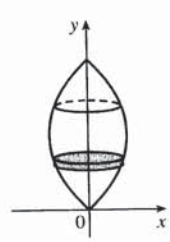


6. A cross-section is a disk with radius $y - y^2$, so its area is $A(y) = \pi\left(y - y^2\right)^2$.

$$V = \int_0^1 A(y)\,dy = \int_0^1 \pi\left(y - y^2\right)^2 dy = \pi\int_0^1 \left(y^4 - 2y^3 + y^2\right) dy = \pi\left[\tfrac{1}{5}y^5 - \tfrac{1}{2}y^4 + \tfrac{1}{3}y^3\right]_0^1$$

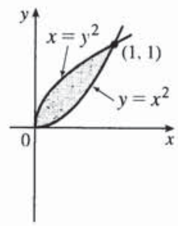
$$= \pi\left(\tfrac{1}{5} - \tfrac{1}{2} + \tfrac{1}{3}\right) = \tfrac{\pi}{30}$$

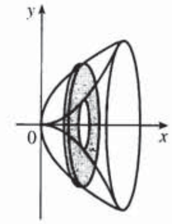




7. A cross-section is a washer (annulus) with inner radius x^2 and outer radius $\sqrt{x}$, so its area is $A(x) = \pi(\sqrt{x}\,)^2 - \pi\left(x^2\right)^2 = \pi\left(x - x^4\right)$.

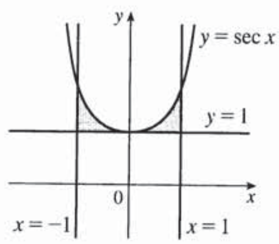
$$V = \int_0^1 A(x)\,dx = \pi\int_0^1 \left(x - x^4\right)dx = \pi\left[\tfrac{1}{2}x^2 - \tfrac{1}{5}x^5\right]_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{5}\right) = \tfrac{3\pi}{10}$$

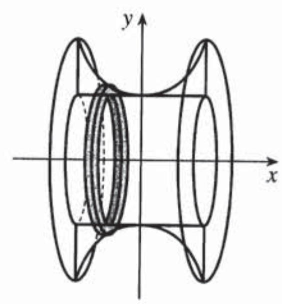




8. A cross-section is a washer with inner radius 1 and outer radius $\sec x$, so its area is $A(x) = \pi(\sec x)^2 - \pi(1)^2 = \pi\left(\sec^2 x - 1\right)$.

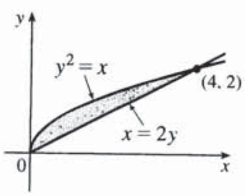
$$V = \int_{-1}^1 A(x)\,dx = \int_{-1}^1 \pi\left(\sec^2 x - 1\right)dx = 2\pi\int_0^1 \left(\sec^2 x - 1\right)dx = 2\pi[\tan x - x]_0^1 = 2\pi(\tan 1 - 1)$$
$$\approx 3.5023$$

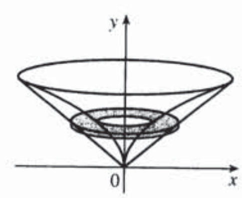




9. A cross-section is a washer with inner radius y^2 and outer radius $2y$, so its area is $A(y) = \pi(2y)^2 - \pi\left(y^2\right)^2 = \pi\left(4y^2 - y^4\right)$.

$$V = \int_0^2 A(y)\,dy = \pi\int_0^2 \left(4y^2 - y^4\right)dy = \pi\left[\tfrac{4}{3}y^3 - \tfrac{1}{5}y^5\right]_0^2 = \pi\left(\tfrac{32}{3} - \tfrac{32}{5}\right) = \tfrac{64\pi}{15}$$

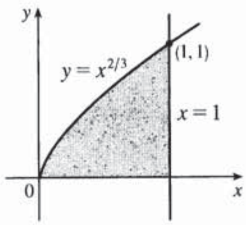


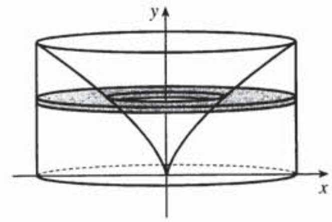


10. $y = x^{2/3} \;\Leftrightarrow\; x = y^{3/2}$, so a cross-section is a washer with inner radius $y^{3/2}$ and outer radius 1, and its area is $A(y) = \pi(1)^2 - \pi\left(y^{3/2}\right)^2 = \pi\left(1 - y^3\right)$.

$$V = \int_0^1 A(y)\,dy = \pi\int_0^1 \left(1 - y^3\right)dy = \pi\left[y - \tfrac{1}{4}y^4\right]_0^1 = \tfrac{3}{4}\pi$$

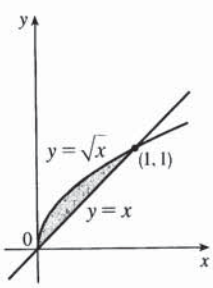
[continued]

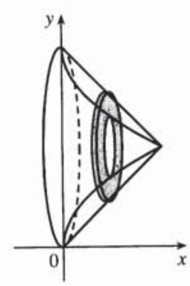




11. A cross-section is a washer with inner radius $1 - \sqrt{x}$ and outer radius $1 - x$, so its area is $A(x) = \pi(1 - x)^2 - \pi(1 - \sqrt{x}\,)^2 = \pi\left[(1 - 2x + x^2) - (1 - 2\sqrt{x} + x)\right] = \pi\left(-3x + x^2 + 2\sqrt{x}\,\right)$.

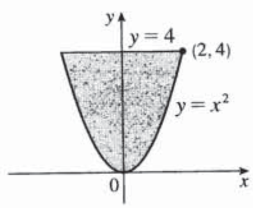
$$\begin{aligned} V &= \int_0^1 A(x)\,dx = \pi \int_0^1 \left(-3x + x^2 + 2\sqrt{x}\,\right) dx \\ &= \pi\left[-\tfrac{3}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{4}{3}x^{3/2}\right]_0^1 = \pi\left(-\tfrac{3}{2} + \tfrac{5}{3}\right) = \tfrac{\pi}{6} \end{aligned}$$

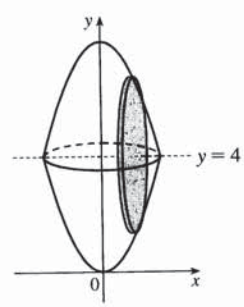




12. A cross-section is circular with radius $4 - x^2$, so its area is $A(x) = \pi\left(4 - x^2\right)^2 = \pi\left(16 - 8x^2 + x^4\right)$.

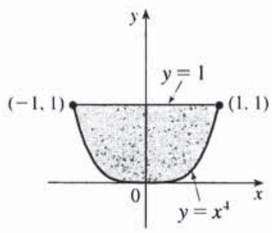
$$\begin{aligned} V &= \int_{-2}^{2} A(x)\,dx = 2\int_0^2 A(x)\,dx = 2\pi \int_0^2 \left(16 - 8x^2 + x^4\right) dx = 2\pi\left[16x - \tfrac{8}{3}x^3 + \tfrac{1}{5}x^5\right]_0^2 \\ &= 2\pi\left(32 - \tfrac{64}{3} + \tfrac{32}{5}\right) = 64\pi\left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right) = 64\pi \cdot \tfrac{8}{15} = \tfrac{512\pi}{15} \end{aligned}$$

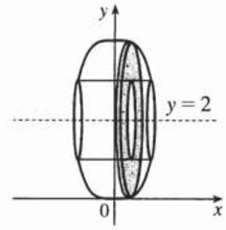




13. A cross-section is an annulus with inner radius $2 - 1$ and outer radius $2 - x^4$, so its area is $A(x) = \pi\left(2 - x^4\right)^2 - \pi(2 - 1)^2 = \pi\left(3 - 4x^4 + x^8\right)$.

$$\begin{aligned} V &= \int_{-1}^{1} A(x)\,dx = 2\int_0^1 A(x)\,dx = 2\pi \int_0^1 \left(3 - 4x^4 + x^8\right) dx = 2\pi\left[3x - \tfrac{4}{5}x^5 + \tfrac{1}{9}x^9\right]_0^1 \\ &= 2\pi\left(3 - \tfrac{4}{5} + \tfrac{1}{9}\right) = \tfrac{208}{45}\pi \end{aligned}$$

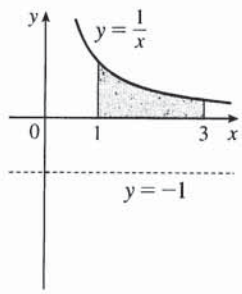


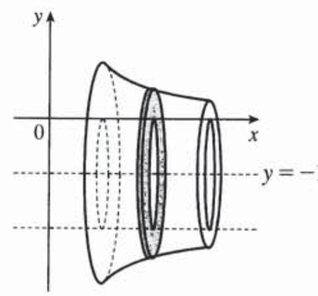


14. $V = \int_1^3 \pi \left\{ \left[\frac{1}{x} - (-1) \right]^2 - [0 - (-1)]^2 \right\} dx = \pi \int_1^3 \left[\left(\frac{1}{x} + 1 \right)^2 - 1^2 \right] dx$

$= \pi \int_1^3 \left(\frac{1}{x^2} + \frac{2}{x} \right) dx = \pi \left[-\frac{1}{x} + 2\ln x \right]_1^3$

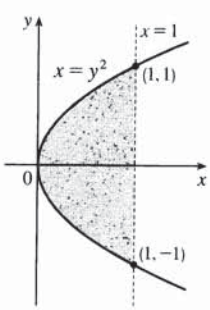
$= \pi\left[\left(-\frac{1}{3} + 2\ln 3\right) - (-1 + 0)\right] = \pi\left(2\ln 3 + \frac{2}{3}\right) = 2\pi\left(\ln 3 + \frac{1}{3}\right)$

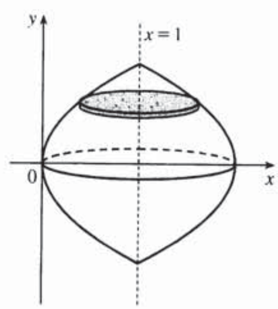




15. $V = \int_{-1}^1 \pi\left(1 - y^2\right)^2 dy = 2\int_0^1 \pi\left(1 - y^2\right)^2 dy = 2\pi \int_0^1 \left(1 - 2y^2 + y^4\right) dy$

$= 2\pi\left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^1 = 2\pi \cdot \frac{8}{15} = \frac{16}{15}\pi$

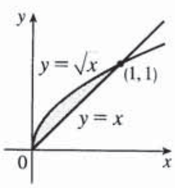


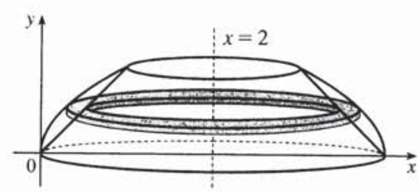


16. $y = \sqrt{x} \quad \Rightarrow \quad x = y^2$, so the outer radius is $2 - y^2$.

$V = \int_0^1 \pi\left[\left(2 - y^2\right)^2 - (2 - y)^2\right] dy = \pi \int_0^1 \left[\left(4 - 4y^2 + y^4\right) - \left(4 - 4y + y^2\right)\right] dy$

$= \pi \int_0^1 \left(y^4 - 5y^2 + 4y\right) dy = \pi\left[\frac{1}{5}y^5 - \frac{5}{3}y^3 + 2y^2\right]_0^1 = \pi\left(\frac{1}{5} - \frac{5}{3} + 2\right) = \frac{8}{15}\pi$



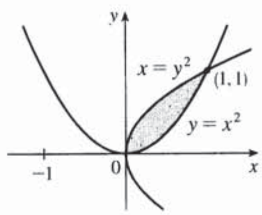


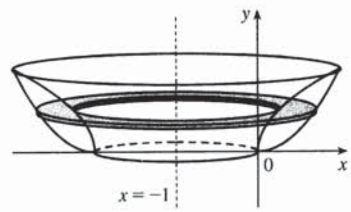
17. $y = x^2 \;\Rightarrow\; x = \sqrt{y}$ for $x \geq 0$. The outer radius is the distance from $x = -1$ to $x = \sqrt{y}$ and the inner radius is the distance from $x = -1$ to $x = y^2$.

$$V = \int_0^1 \pi \left\{ \left[\sqrt{y} - (-1)\right]^2 - \left[y^2 - (-1)\right]^2 \right\} dy = \pi \int_0^1 \left[\left(\sqrt{y} + 1\right)^2 - \left(y^2 + 1\right)^2 \right] dy$$

$$= \pi \int_0^1 \left(y + 2\sqrt{y} + 1 - y^4 - 2y^2 - 1\right) dy = \pi \int_0^1 \left(y + 2\sqrt{y} - y^4 - 2y^2\right) dy$$

$$= \pi \left[\tfrac{1}{2}y^2 + \tfrac{4}{3}y^{3/2} - \tfrac{1}{5}y^5 - \tfrac{2}{3}y^3 \right]_0^1 = \pi\left(\tfrac{1}{2} + \tfrac{4}{3} - \tfrac{1}{5} - \tfrac{2}{3}\right) = \tfrac{29}{30}\pi$$



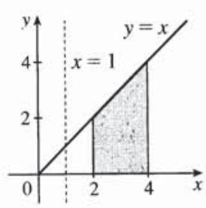


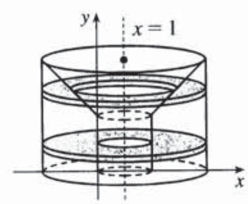
18. For $0 \leq y < 2$, a cross-section is an annulus with inner radius $2 - 1$ and outer radius $4 - 1$, the area of which is $A_1(y) = \pi(4-1)^2 - \pi(2-1)^2$. For $2 \leq y \leq 4$, a cross-section is an annulus with inner radius $y - 1$ and outer radius $4 - 1$, the area of which is $A_2(y) = \pi(4-1)^2 - \pi(y-1)^2$.

$$V = \int_0^4 A(y)\,dy = \pi \int_0^2 \left[(4-1)^2 - (2-1)^2\right] dy + \pi \int_2^4 \left[(4-1)^2 - (y-1)^2\right] dy$$

$$= \pi\left[8y\right]_0^2 + \pi \int_2^4 \left(8 + 2y - y^2\right) dy = 16\pi + \pi\left[8y + y^2 - \tfrac{1}{3}y^3\right]_2^4$$

$$= 16\pi + \pi\left[\left(32 + 16 - \tfrac{64}{3}\right) - \left(16 + 4 - \tfrac{8}{3}\right)\right] = \tfrac{76}{3}\pi$$





19. $\mathcal{R}_1$ about OA (the line $y = 0$): $V = \int_0^1 A(x)\,dx = \int_0^1 \pi(x^3)^2\,dx = \pi \int_0^1 x^6\,dx = \pi\left[\tfrac{1}{7}x^7\right]_0^1 = \tfrac{\pi}{7}$

20. $\mathcal{R}_1$ about OC (the line $x = 0$):

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left[\pi(1)^2 - \pi\left(\sqrt[3]{y}\right)^2\right] dy = \pi \int_0^1 (1 - y^{2/3})\,dy = \pi\left[y - \tfrac{3}{5}y^{5/3}\right]_0^1 = \pi\left(1 - \tfrac{3}{5}\right) = \tfrac{2\pi}{5}$$

21. $\mathcal{R}_1$ about AB (the line $x = 1$):

$$V = \int_0^1 A(y)\,dy = \int_0^1 \pi\left(1 - \sqrt[3]{y}\right)^2 dy = \pi \int_0^1 (1 - 2y^{1/3} + y^{2/3})\,dy$$

$$= \pi\left[y - \tfrac{3}{2}y^{4/3} + \tfrac{3}{5}y^{5/3}\right]_0^1 = \pi\left(1 - \tfrac{3}{2} + \tfrac{3}{5}\right) = \tfrac{\pi}{10}$$

22. $\mathcal{R}_1$ about BC (the line $y = 1$):

$$V = \int_0^1 A(x)\,dx = \int_0^1 \left[\pi(1)^2 - \pi(1 - x^3)^2\right] dx = \pi \int_0^1 \left[1 - (1 - 2x^3 + x^6)\right] dx$$

$$= \pi \int_0^1 (2x^3 - x^6)\,dx = \pi\left[\tfrac{1}{2}x^4 - \tfrac{1}{7}x^7\right]_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{7}\right) = \tfrac{5\pi}{14}$$

23. $\mathcal{R}_2$ about OA (the line $y = 0$):

$$V = \int_0^1 A(x)\,dx = \int_0^1 \left[\pi(1)^2 - \pi\left(\sqrt{x}\right)^2\right] dx = \pi \int_0^1 (1 - x)\,dx = \pi\left[x - \tfrac{1}{2}x^2\right]_0^1 = \pi\left(1 - \tfrac{1}{2}\right) = \tfrac{\pi}{2}$$

24. $\mathcal{R}_2$ about OC (the line $x = 0$): $V = \int_0^1 A(y)\,dy = \int_0^1 \pi(y^2)^2\,dy = \pi \int_0^1 y^4\,dy = \pi\left[\frac{1}{5}y^5\right]_0^1 = \frac{\pi}{5}$

25. $\mathcal{R}_2$ about AB (the line $x = 1$):

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left[\pi(1)^2 - \pi(1 - y^2)^2\right] dy = \pi \int_0^1 \left[1 - (1 - 2y^2 + y^4)\right] dy$$
$$= \pi \int_0^1 (2y^2 - y^4)\,dy = \pi\left[\tfrac{2}{3}y^3 - \tfrac{1}{5}y^5\right]_0^1 = \pi\left(\tfrac{2}{3} - \tfrac{1}{5}\right) = \tfrac{7\pi}{15}$$

26. $\mathcal{R}_2$ about BC (the line $y = 1$):

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi(1 - \sqrt{x})^2\,dx = \pi \int_0^1 (1 - 2x^{1/2} + x)\,dx$$
$$= \pi\left[x - \tfrac{4}{3}x^{3/2} + \tfrac{1}{2}x^2\right]_0^1 = \pi\left(1 - \tfrac{4}{3} + \tfrac{1}{2}\right) = \tfrac{\pi}{6}$$

27. $\mathcal{R}_3$ about OA (the line $y = 0$):

$$V = \int_0^1 A(x)\,dx = \int_0^1 \left[\pi(\sqrt{x})^2 - \pi(x^3)^2\right] dx = \pi \int_0^1 (x - x^6)\,dx = \pi\left[\tfrac{1}{2}x^2 - \tfrac{1}{7}x^7\right]_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{7}\right) = \tfrac{5\pi}{14}.$$

Note: Let $\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3$. If we rotate $\mathcal{R}$ about any of the segments OA, OC, AB, or BC, we obtain a right circular cylinder of height 1 and radius 1. Its volume is $\pi r^2 h = \pi(1)^2 \cdot 1 = \pi$. As a check for Exercises 19, 23, and 27, we can add the answers, and that sum must equal π. Thus, $\frac{\pi}{7} + \frac{\pi}{2} + \frac{5\pi}{14} = \left(\frac{2+7+5}{14}\right)\pi = \pi$.

28. $\mathcal{R}_3$ about OC (the line $x = 0$):

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left[\pi\left(\sqrt[3]{y}\right)^2 - \pi(y^2)^2\right] dy = \pi \int_0^1 (y^{2/3} - y^4)\,dy$$
$$= \pi\left[\tfrac{3}{5}y^{5/3} - \tfrac{1}{5}y^5\right]_0^1 = \pi\left(\tfrac{3}{5} - \tfrac{1}{5}\right) = \tfrac{2\pi}{5}$$

Note: See the note in Exercise 27. For Exercises 20, 24, and 28, we have $\frac{2\pi}{5} + \frac{\pi}{5} + \frac{2\pi}{5} = \pi$.

29. $\mathcal{R}_3$ about AB (the line $x = 1$):

$$V = \int_0^1 A(y)\,dy = \int_0^1 \left[\pi(1 - y^2)^2 - \pi\left(1 - \sqrt[3]{y}\right)^2\right] dy = \pi \int_0^1 \left[(1 - 2y^2 + y^4) - (1 - 2y^{1/3} + y^{2/3})\right] dy$$
$$= \pi \int_0^1 (-2y^2 + y^4 + 2y^{1/3} - y^{2/3})\,dy = \pi\left[-\tfrac{2}{3}y^3 + \tfrac{1}{5}y^5 + \tfrac{3}{2}y^{4/3} - \tfrac{3}{5}y^{5/3}\right]_0^1$$
$$= \pi\left(-\tfrac{2}{3} + \tfrac{1}{5} + \tfrac{3}{2} - \tfrac{3}{5}\right) = \tfrac{13\pi}{30}$$

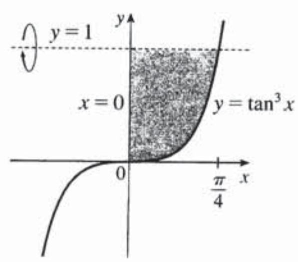
Note: See the note in Exercise 27. For Exercises 21, 25, and 29, we have $\frac{\pi}{10} + \frac{7\pi}{15} + \frac{13\pi}{30} = \left(\frac{3+14+13}{30}\right)\pi = \pi$.

30. $\mathcal{R}_3$ about BC (the line $y = 1$):

$$V = \int_0^1 A(x)\,dx = \int_0^1 \left[\pi(1 - x^3)^2 - \pi(1 - \sqrt{x})^2\right] dx$$
$$= \pi \int_0^1 \left[(1 - 2x^3 + x^6) - (1 - 2x^{1/2} + x)\right] dx = \pi \int_0^1 (-2x^3 + x^6 + 2x^{1/2} - x)\,dx$$
$$= \pi\left[-\tfrac{1}{2}x^4 + \tfrac{1}{7}x^7 + \tfrac{4}{3}x^{3/2} - \tfrac{1}{2}x^2\right]_0^1 = \pi\left(-\tfrac{1}{2} + \tfrac{1}{7} + \tfrac{4}{3} - \tfrac{1}{2}\right) = \tfrac{10\pi}{21}$$

Note: See the note in Exercise 27. For Exercises 22, 26, and 30, we have $\frac{5\pi}{14} + \frac{\pi}{6} + \frac{10\pi}{21} = \left(\frac{15+7+20}{42}\right)\pi = \pi$.

31. $V = \pi \int_0^{\pi/4} (1 - \tan^3 x)^2\,dx$



32. $y = (x-2)^4$ and $8x - y = 16$ intersect when

$(x-2)^4 = 8x - 16 = 8(x-2) \quad \Leftrightarrow$

$(x-2)^4 - 8(x-2) = 0 \quad \Leftrightarrow \quad (x-2)\left[(x-2)^3 - 8\right] = 0$

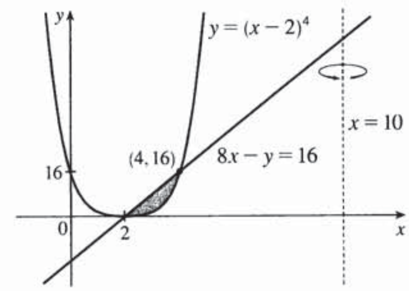
$\Leftrightarrow \quad x - 2 = 0 \quad \text{or} \quad x - 2 = 2 \quad \Leftrightarrow \quad x = 2 \text{ or } 4.$

$y = (x-2)^4 \quad \Rightarrow \quad x - 2 = \pm\sqrt[4]{y} \quad \Rightarrow \quad x = 2 + \sqrt[4]{y}$

[since $x \geq 2$]. $8x - y = 16 \quad \Rightarrow \quad 8x = y + 16 \quad \Rightarrow$

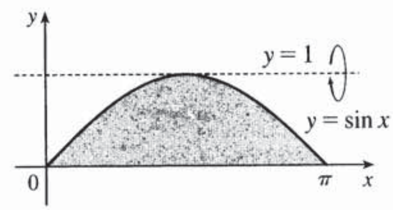
$x = \frac{1}{8}y + 2.$

$$V = \pi \int_0^{16} \left\{ \left[10 - \left(\tfrac{1}{8}y + 2\right)\right]^2 - \left[10 - (2 + \sqrt[4]{y})\right]^2 \right\} dy$$



33. $$V = \pi \int_0^{\pi} \left[(1-0)^2 - (1 - \sin x)^2\right] dx$$

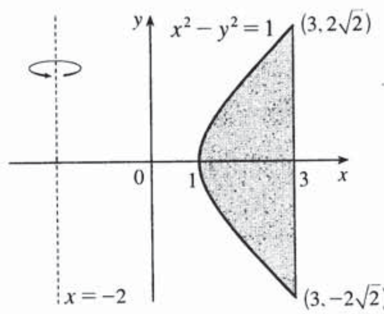
$$= \pi \int_0^{\pi} \left[1^2 - (1 - \sin x)^2\right] dx$$



34. $V = \pi \int_0^{\pi} \left[(\sin x + 2)^2 - 2^2\right] dx$

35. $$V = \pi \int_{-\sqrt{8}}^{\sqrt{8}} \left\{ [3 - (-2)]^2 - \left[\sqrt{y^2+1} - (-2)\right]^2 \right\} dy$$

$$= \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} \left[5^2 - \left(\sqrt{1+y^2} + 2\right)^2\right] dy$$

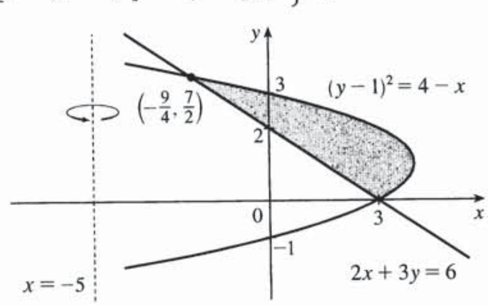


36. Solve the equations for x: $(y-1)^2 = 4 - x \quad \Leftrightarrow \quad x = 4 - (y-1)^2$ and $2x + 3y = 6 \quad \Leftrightarrow \quad x = 3 - \frac{3}{2}y$.

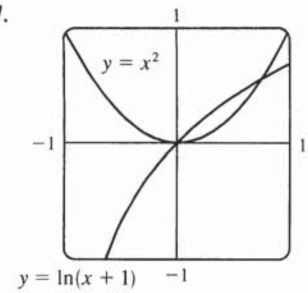
The points of intersection of the two curves are $(3, 0)$ and $\left(-\frac{9}{4}, \frac{7}{2}\right)$. Therefore,

$$V = \pi \int_0^{7/2} \left\{ \left[4 - (y-1)^2 - (-5)\right]^2 - \left[3 - \tfrac{3}{2}y - (-5)\right]^2 \right\} dy$$

$$= \pi \int_0^{7/2} \left\{ \left[9 - (y-1)^2\right]^2 - \left(8 - \tfrac{3}{2}y\right)^2 \right\} dy$$



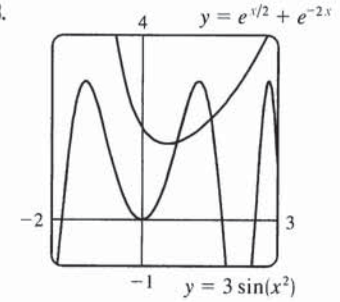
37.



$y = x^2$ and $y = \ln(x+1)$ intersect at $x = 0$ and at $x = a \approx 0.747$.

$$V = \pi \int_0^a \left\{ [\ln(x+1)]^2 - \left(x^2\right)^2 \right\} dx \approx 0.132$$

38.

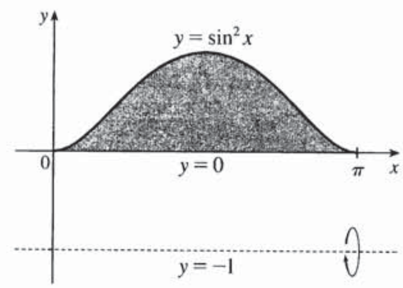


$y = 3\sin(x^2)$ and $y = e^{x/2} + e^{-2x}$ intersect at $x = a \approx 0.772$ and at $x = b \approx 1.524$.

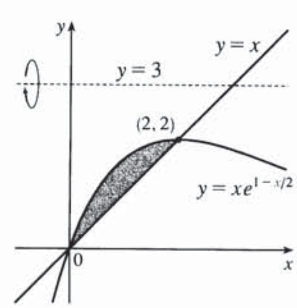
$$V = \pi \int_a^b \left\{ \left[3\sin(x^2)\right]^2 - \left(e^{x/2} + e^{-2x}\right)^2 \right\} dx$$

$$\approx 7.519$$

39. $V = \pi \displaystyle\int_0^\pi \left\{ \left[\sin^2 x - (-1)\right]^2 - [0 - (-1)]^2 \right\} dx \overset{\text{CAS}}{=} \frac{11}{8}\pi^2$



40. $V = \pi \displaystyle\int_0^2 \left[(3-x)^2 - (3 - xe^{1-x/2})^2 \right] dx \overset{\text{CAS}}{=} \pi\left(-2e^2 + 24e - \frac{142}{3}\right)$



41. $\pi \int_0^{\pi/2} \cos^2 x\, dx$ describes the volume of the solid obtained by rotating the region $\mathscr{R} = \left\{(x,y) \mid 0 \le x \le \frac{\pi}{2}, 0 \le y \le \cos x\right\}$ of the xy-plane about the x-axis.

42. $\pi \int_2^5 y\, dy = \pi \int_2^5 \left(\sqrt{y}\right)^2 dy$ describes the volume of the solid obtained by rotating the region $\mathscr{R} = \left\{(x,y) \mid 2 \le y \le 5, 0 \le x \le \sqrt{y}\right\}$ of the xy-plane about the y-axis.

43. $\pi \displaystyle\int_0^1 \left(y^4 - y^8\right) dy = \pi \int_0^1 \left[\left(y^2\right)^2 - \left(y^4\right)^2\right] dy$ describes the volume of the solid obtained by rotating the region $\mathscr{R} = \left\{(x,y) \mid 0 \le y \le 1, y^4 \le x \le y^2\right\}$ of the xy-plane about the y-axis.