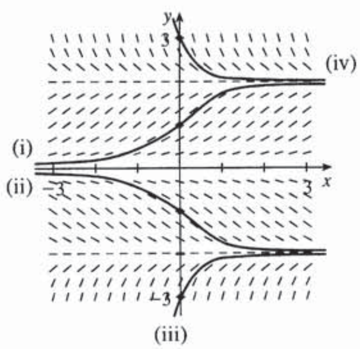
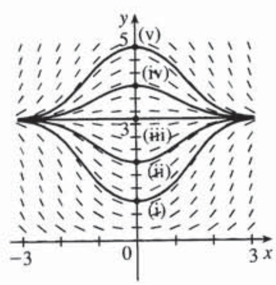
9.2 Direction Fields and Euler's Method

1. (a)



(b) It appears that the constant functions $y = 0$, $y = -2$, and $y = 2$ are equilibrium solutions. Note that these three values of y satisfy the given differential equation $y' = y\left(1 - \frac{1}{4}y^2\right)$.

2. (a)



(b) From the figure, it appears that $y = \pi$ is an equilibrium solution. From the equation $y' = x \sin y$, we see that $y = n\pi$ (n an integer) describes all the equilibrium solutions.

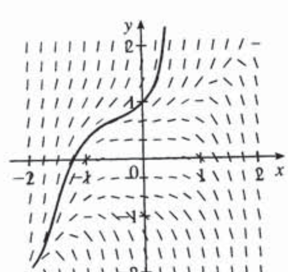
3. $y' = y - 1$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, IV is the direction field for this equation. Note that for $y = 1$, $y' = 0$.

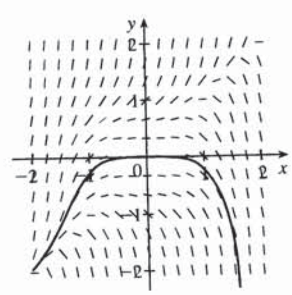
4. $y' = y - x = 0$ on the line $y = x$, when $x = 0$ the slope is y, and when $y = 0$ the slope is $-x$. Direction field II satisfies these conditions. [Looking at the slope at the point $(0, 2)$, II looks more like it has a slope of 2 than does direction field I.]

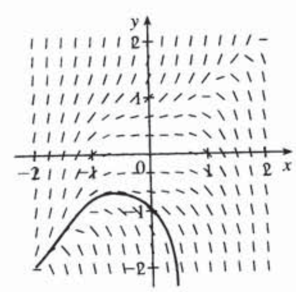
5. $y' = y^2 - x^2 = 0 \quad \Rightarrow \quad y = \pm x$. There are horizontal tangents on these lines only in graph III, so this equation corresponds to direction field III.

6. $y' = y^3 - x^3 = 0$ on the line $y = x$, when $x = 0$ the slope is y^3, and when $y = 0$ the slope is $-x^3$. The graph is similar to the graph for Exercise 4, but the segments must get steeper very rapidly as they move away from the origin, because x and y are raised to the third power. This is the case in direction field I.

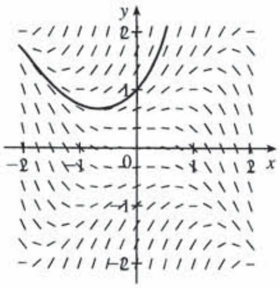
7. (a) $y(0) = 1$ (b) $y(0) = 0$ (c) $y(0) = -1$

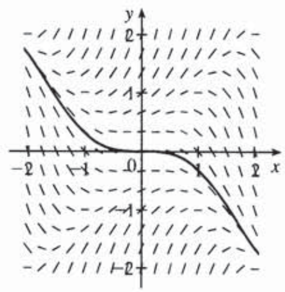


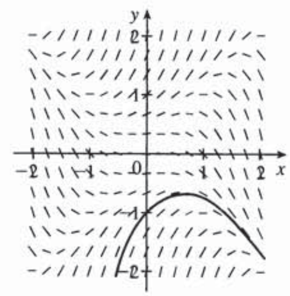




8. (a) $y(0) = 1$ (b) $y(0) = 0$ (c) $y(0) = -1$



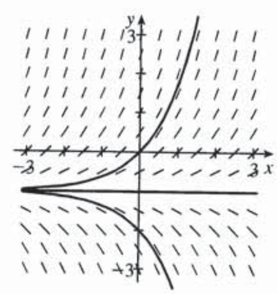




9.

x	y	$y' = 1 + y$
0	0	1
0	1	2
0	2	3
0	-3	-2
0	-2	-1

Note that for $y = -1$, $y' = 0$. The three solution curves sketched go through $(0, 0)$, $(0, -1)$, and $(0, -2)$.



10.

x	y	$y' = x^2 - y^2$
± 1	± 3	-8
± 3	± 1	8
± 1	± 0.5	0.75
± 0.5	± 1	-0.75

Note that $y' = 0$ for $y = \pm x$. If $|x| < |y|$, then $y' < 0$; that is, the slopes are negative for all points in quadrants I and II above both of the lines $y = x$ and $y = -x$, and all points in quadrants III and IV below both of the lines $y = -x$ and $y = x$. A similar statement holds for positive slopes.