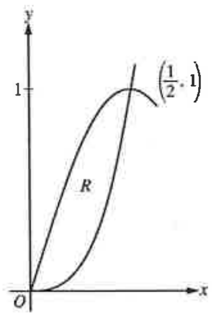
CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

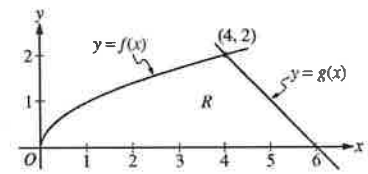
   (a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

   (b) Find the area of R.

   (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

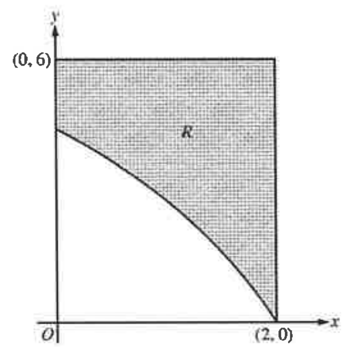
   (a) Find the area of R.

   (b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

   (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

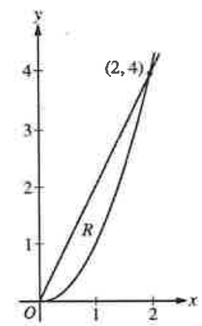
A graphing calculator is required for some problems or parts of problems.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

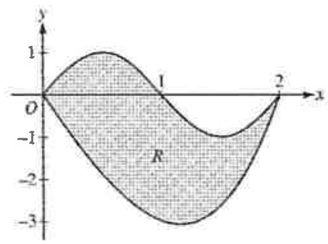
No calculator is allowed for these problems.



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.
   (a) Find the area of R.
   (b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
   (c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.
   (a) Find the area of R.
   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.
   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.
   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

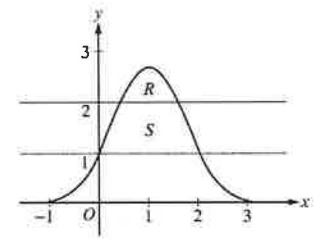
(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

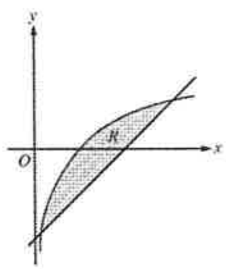
(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

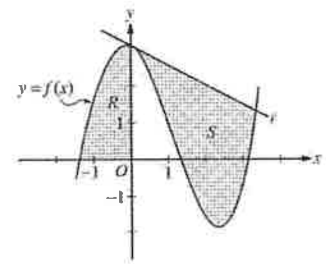
   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

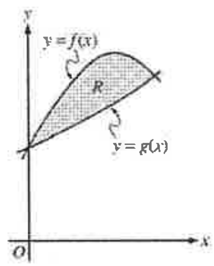
   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

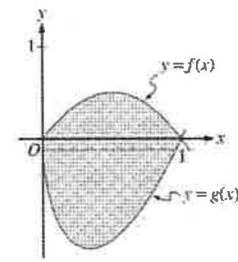


1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



2. Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

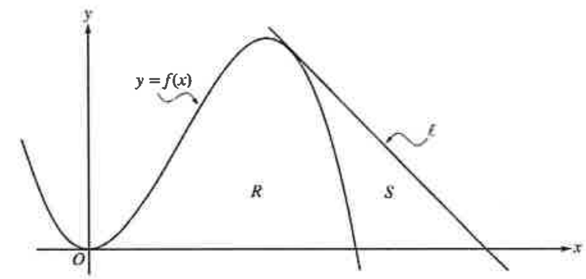
   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

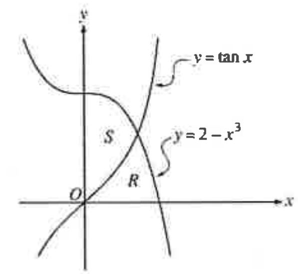
(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.

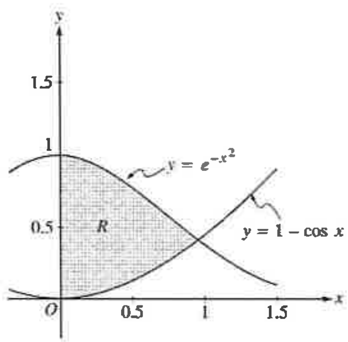
(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



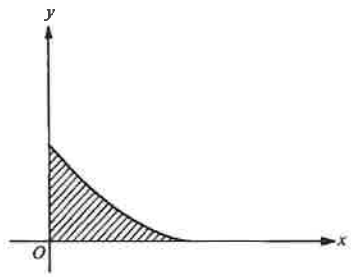
1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

(a) Find the area of R.

(b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k ?

(c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

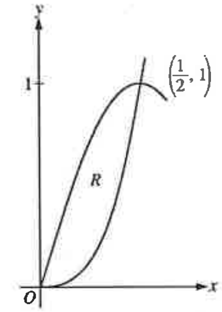


The base of a solid S is the shaded region in the first quadrant enclosed by the coordinate axes and the graph of $y = 1 - \sin x$, as shown in the figure above. For each x, the cross section of S perpendicular to the x-axis at the point $(x, 0)$ is an isosceles right triangle whose hypotenuse lies in the xy-plane.

(a) Find the area of the triangle as a function of x.

(b) Find the volume of S.

Question 3



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

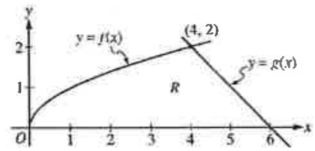
(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$ $f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$ An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.	2 : { 1 : $f'\left(\frac{1}{2}\right)$; 1 : answer }
(b) Area $= \int_0^{1/2} (g(x) - f(x))\,dx$ $= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx$ $= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2}$ $= -\frac{1}{8} + \frac{1}{\pi}$	4 : { 1 : integrand; 2 : antiderivative; 1 : answer }
(c) $\pi\int_0^{1/2}\left((1 - f(x))^2 - (1 - g(x))^2\right) dx$ $= \pi\int_0^{1/2}\left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx$	3 : { 1 : limits and constant; 2 : integrand }

Question 3

The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

(a) Area $= \int_0^4 \sqrt{x}\,dx + \frac{1}{2}\cdot 2\cdot 2 = \frac{2}{3}x^{3/2}\Big|_{x=0}^{x=4} + 2 = \frac{22}{3}$

$3: \begin{cases} 1: \text{integral} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(b) $y = \sqrt{x} \Rightarrow x = y^2$

$y = 6 - x \Rightarrow x = 6 - y$

Width $= (6 - y) - y^2$

Volume $= \int_0^2 2y\left(6 - y - y^2\right) dy$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

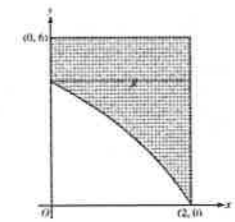
$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3: \begin{cases} 1: f'(x) \\ 1: \text{equation} \\ 1: \text{answer} \end{cases}$

Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

1 : Correct limits in an integral in (a), (b), or (c)

(a) $\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or 6.817

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) $\pi\int_0^2 \left((8 - 4\ln(3 - x))^2 - (8 - 6)^2\right) dx$

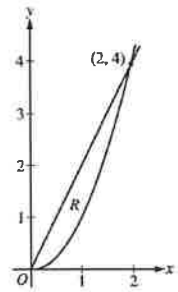
$= 168.179$ or 168.180

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $\int_0^2 (6 - 4\ln(3 - x))^2\,dx = 26.266$ or 26.267

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

Question 4



Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

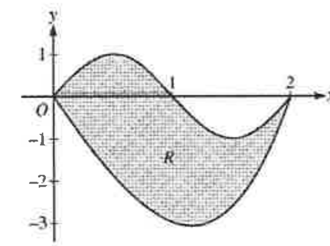
(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

(a) Area $= \int_0^2 (2x - x^2)\,dx$ $= x^2 - \frac{1}{3}x^3\Big\|_{x=0}^{x=2}$ $= \frac{4}{3}$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx$ $= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big\|_{x=0}^{x=2}$ $= \frac{4}{\pi}$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(c) Volume $= \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits} \end{cases}$

Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - (x^3 - 4x)\right) dx = 4$	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - (x^3 - 4x)\right) dx$	$2: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - (x^3 - 4x)\right)^2 dx = 9.978$	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - (x^3 - 4x)\right) dx = 8.369$ or 8.370	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$

$4: \begin{cases} 1: \text{constant and limits} \\ 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and answer} \end{cases}$

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$\frac{20}{1+x^2} = 2$ when $x = \pm 3$

1 : correct limits in an integral in (a), (b), or (c)

(a) Area $= \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right) dx = 37.961 \text{ or } 37.962$

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) Volume $= \pi \int_{-3}^{3} \left(\left(\frac{20}{1+x^2}\right)^2 - 2^2\right) dx = 1871.190$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

(c) Volume $= \frac{\pi}{2} \int_{-3}^{3} \left(\frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)\right)^2 dx$

$= \frac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right)^2 dx = 174.268$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

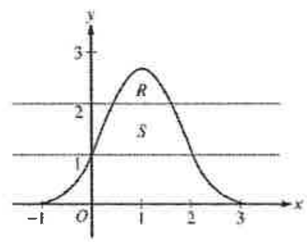
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx -$ Area of R
$= 2.06016 -$ Area of $R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q-P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$
$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{constant and limits} \end{cases}$

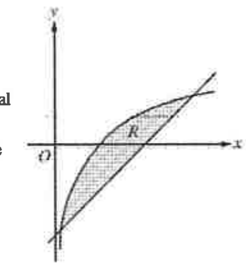
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x-2))\, dx = 1.949$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x)+3)^2 - (x-2+3)^2\right) dx$
$= 34.198$ or 34.199

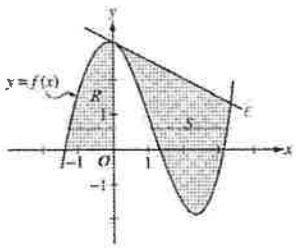
$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - (e^y)^2\right) dy$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$

Question 1

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.



(a) Find the area of R.

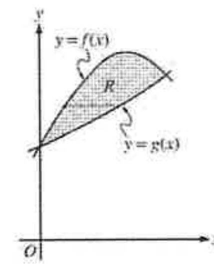
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

For $x < 0$, $f(x) = 0$ when $x = -1.37312$. Let $P = -1.37312$.	
(a) Area of $R = \int_P^0 f(x)\,dx = 2.903$	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi\int_P^0 \left((f(x)+2)^2 - 4\right)dx = 59.361$	$4: \begin{cases} 1: \text{limits and constant} \\ 2: \text{integrand} \\ 1: \text{answer} \end{cases}$
(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$. The graph of f and line ℓ intersect at $A = 3.38987$. Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x\right) - f(x)\right)dx$	$3: \begin{cases} 1: \text{tangent line} \\ 1: \text{integrand} \\ 1: \text{limits} \end{cases}$

Question 1

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R.

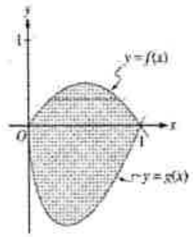
(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_0^S (f(x) - g(x))\,dx$ $= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right)dx$ $= 0.429$	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi\int_0^S \left((f(x))^2 - (g(x))^2\right)dx$ $= \pi\int_0^S \left((1+\sin(2x))^2 - \left(e^{x/2}\right)^2\right)dx$ $= 4.266$ or 4.267	$3: \begin{cases} 2: \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: 0/2 if integral not of form} \\ \quad c\int_a^b \left(R^2(x) - r^2(x)\right)dx \\ 1: \text{answer} \end{cases}$
(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx$ $= \int_0^S \frac{\pi}{2}\left(\frac{1+\sin(2x) - e^{x/2}}{2}\right)^2 dx$ $= 0.077$ or 0.078	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

Question 2

Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.



(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) Area $= \int_0^1 (f(x) - g(x))\,dx$

$= \int_0^1 (2x(1-x) - 3(x-1)\sqrt{x})\,dx = 1.133$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^1 \left((2-g(x))^2 - (2-f(x))^2\right)dx$

$= \pi\int_0^1 \left((2-3(x-1)\sqrt{x})^2 - (2-2x(1-x))^2\right)dx$

$= 16.179$

$4: \begin{cases} 1: \text{limits and constant} \\ 2: \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: 0/2 if integral not of form} \\ \quad\quad c\int_a^b \left(R^2(x) - r^2(x)\right)dx \\ 1: \text{answer} \end{cases}$

(c) Volume $= \int_0^1 (h(x) - g(x))^2\,dx$

$\int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx = 15$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

Question 1

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

(a) Area $= \int_1^{10} \sqrt{x-1}\,dx = 18$

$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) Volume $= \pi\int_1^{10} \left(9 - (3-\sqrt{x-1})^2\right)dx$

$= 212.057$ or 212.058

$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

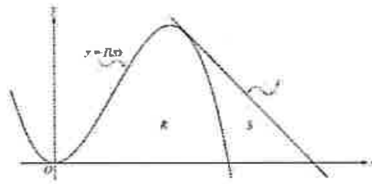
(c) Volume $= \pi\int_0^3 \left(10 - (y^2+1)\right)^2 dy$

$= 407.150$

$3: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.



(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$

$f(3) = 36 - 27 = 9$

Tangent line at $x = 3$ is

$y = -3(x-3) + 9 = -3x + 18$,

which is the equation of line ℓ.

2 :
- 1 : finds $f'(3)$ and $f(3)$
- 1 : finds equation of tangent line or shows (3,9) is on both the graph of f and line ℓ

(b) $f(x) = 0$ at $x = 4$

The line intersects the x-axis at $x = 6$.

Area $= \frac{1}{2}(3)(9) - \int_3^4 (4x^2 - x^3)\,dx$

$= 7.916$ or 7.917

OR

Area $= \int_3^4 \left((18 - 3x) - (4x^2 - x^3)\right)dx + \frac{1}{2}(2)(18 - 12)$

$= 7.916$ or 7.917

4 :
- 2 : integral for non-triangular region
  - 1 : limits
  - 1 : integrand
- 1 : area of triangular region
- 1 : answer

(c) Volume $= \pi \int_0^4 (4x^2 - x^3)^2\,dx$

$= 156.038\pi$ or 490.208

3 :
- 1 : limits and constant
- 1 : integrand
- 1 : answer

Question 1

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) Area $= \int_{1/2}^{1} (e^x - \ln x)\,dx = 1.222$ or 1.223

2 :
- 1 : integral
- 1 : answer

(b) Volume $= \pi \int_{1/2}^{1} \left((4 - \ln x)^2 - (4 - e^x)^2\right)dx$

$= 7.515\pi$ or 23.609

4 :
- 1 : limits and constant
- 2 : integrand
  - $<-1>$ each error
  - Note: 0/2 if not of the form $k\int_a^b \left(R(x)^2 - r(x)^2\right)dx$
- 1 : answer

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0$

$x = 0.567143$

Absolute minimum value and absolute maximum value occur at the critical point or at the endpoints.

$h(0.567143) = 2.330$

$h(0.5) = 2.3418$

$h(1) = 2.718$

The absolute minimum is 2.330.

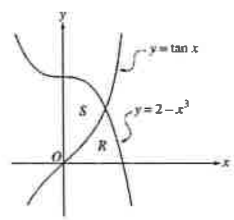
The absolute maximum is 2.718.

3 :
- 1 : considers $h'(x) = 0$
- 1 : identifies critical point and endpoints as candidates
- 1 : answers

Note: Errors in computation come off the third point.

Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



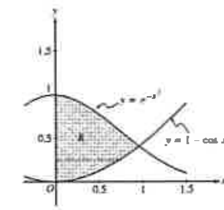
(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.

Point of intersection $2 - x^3 = \tan x$ at $(A, B) = (0.902155, 1.265751)$	
(a) Area $R = \int_0^A \tan x\,dx + \int_A^{\sqrt[3]{2}} (2 - x^3)\,dx = 0.729$ or Area $R = \int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy = 0.729$ or Area $R = \int_0^{\sqrt[3]{2}} (2 - x^3)\,dx - \int_0^A (2 - x^3 - \tan x)\,dx = 0.729$	$3: \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Area $S = \int_0^A (2 - x^3 - \tan x)\,dx = 1.160$ or 1.161 or Area $S = \int_0^B \tan^{-1} y\,dy + \int_B^2 (2-y)^{1/3}\,dy = 1.160$ or 1.161 or Area S $= \int_0^2 (2-y)^{1/3}\,dy - \int_0^B \left((2-y)^{1/3} - \tan^{-1} y\right) dy$ $= 1.160$ or 1.161	$3: \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \pi \int_0^A \left((2 - x^3)^2 - \tan^2 x\right) dx$ $= 2.652\pi$ or 8.331 or 8.332	$3: \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.



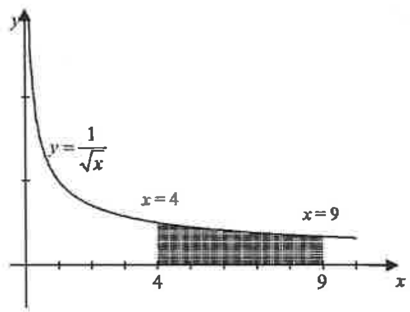
(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R $e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$ $= 0.590$ or 0.591	$2 \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right) dx$ $= 0.55596\pi = 1.746$ or 1.747	$3 \begin{cases} 2: \text{integrand and constant} \\ \quad <-1> \text{ each error} \\ 1: \text{answer} \end{cases}$
(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$ $= 0.461$	$3 \begin{cases} 2: \text{integrand} \\ \quad <-1> \text{ each error} \\ \quad \text{Note: 0/2 if not of the form} \\ \quad k\int_c^d (f(x) - g(x))^2\,dx \\ 1: \text{answer} \end{cases}$

(a) $\int_4^9 \frac{dx}{\sqrt{x}} dx = 2$



(b) $\int_4^k \frac{dx}{\sqrt{x}} dx = 1$

$2\sqrt{x}\Big]_4^k = 1$

$2\sqrt{k} - 2\sqrt{4} = 1$

$k = \frac{25}{4}$

$\left(\text{or } \int_k^9 \frac{dx}{\sqrt{x}} = 1 \text{ or } \int_4^k \frac{dx}{\sqrt{x}} = \int_k^9 \frac{dx}{\sqrt{x}}\right)$

(c) $\text{Volume} = \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$

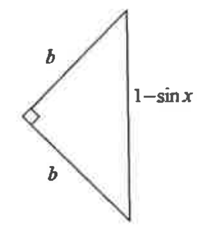
$= \int_4^9 \frac{dx}{x} = \ln x\Big|_4^9 = \ln\frac{9}{4}$ (or 0.811)

(a) $2b^2 = (1 - \sin x)^2$

$b^2 = \frac{1}{2}(1 - \sin x)^2$

$\text{Area} = \frac{1}{2}b^2$

$A(x) = \frac{1}{4}(1 - \sin x)^2$



(b) $\text{Volume} = \int_0^{\pi/2} \frac{1}{4}(1 - \sin x)^2 dx$

$= \frac{1}{4}\int_0^{\pi/2}\left(1 - 2\sin x + \frac{1}{2}(1 - \cos 2x)\right)dx$

$= \frac{1}{4}\left(x + 2\cos x + \frac{1}{2}x - \frac{1}{4}\sin 2x\right)\Big|_0^{\pi/2}$

$= \frac{1}{4}\left(\left(\frac{\pi}{2} + \frac{\pi}{4}\right) - 2\right)$

$= \frac{3\pi}{16} - \frac{1}{2}$