(29) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx =$

$u = \sin x$
$du = \cos x\,dx$
$dx = \frac{du}{\cos x}$

$\sin \pi/2 = 1$
$\sin \pi/4 = \sqrt{2}/2$

(1)

$\int \frac{\cos x}{u} \frac{du}{\cos x}$

$\int_{\sqrt{2}/2}^{1} \frac{1}{u}\,du = \ln u \Big|_{\sqrt{2}/2}^{1} = \ln 1 - \ln \frac{\sqrt{2}}{2} = 0 - \ln \frac{1}{2}\sqrt{2} = \ln\left(\frac{\sqrt{2}}{2}\right)^{-1}$

$= \ln \frac{2}{\sqrt{2}} = \ln \sqrt{2}$

(38) $\int \frac{x^2}{e^{x^3}}\,dx$

$u = e^{x^3}$
$du = e^{x^3} \cdot 3x^2\,dx$
$\frac{du}{e^{x^3} \cdot 3x^2} = dx$

(2)

$\int \frac{x^2}{u} \cdot \frac{du}{e^{x^3} 3x^2} =$

$\frac{1}{3}\int \frac{1}{u} \cdot \frac{du}{u} =$

$\frac{1}{3}\int u^{-2}\,du =$

$\frac{1}{3} \frac{u^{-1}}{-1} + C =$

$-\frac{1}{3u} = -\frac{1}{3e^{x^3}} + C$

(43) $\int \sin(2x+3)\,dx$

$u = 2x+3x$

$du = 2\,dx$

$dx = \frac{du}{2}$

(3)

$= \int \sin u \cdot \frac{du}{2}$

$= \frac{1}{2}\int \sin u \, du$

$= -\frac{1}{2} \cos u + C$

$= -\frac{1}{2} \cos(2x+3) + C$

(21) $\int_0^1 (x+1)\, e^{x^2+2x}\,dx$

$u = e^{x^2+2x}$

$du = e^{x^2+2x} \cdot (2x+2)\,dx$

$\frac{du}{e^{x^2+2x}(2x+2)} = dx$

(4)

$\int (x+1)\, u \cdot \frac{du}{e^{x^2+2x}(2x+2)}$

$= \int_1^{e^3} (x+1)\, u \, \frac{du}{e^{x^2+2x} \cdot 2(x+1)} = \frac{1}{2}\int u \, \frac{du}{u} = \frac{1}{2}\int du$

$= \frac{1}{2} u \Big|_1^{e^3} \qquad \frac{1}{2}e^3 - \frac{1}{2} \cdot 1 \qquad = \frac{1}{2}e^3 - \frac{1}{2}$

$= \frac{e^3 - 1}{2}$

(5)

(27) $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx$

$u = \sqrt{1-x^2} = (1-x^2)^{1/2}$

$du = \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x\,dx$

$du = \frac{-x}{\sqrt{1-x^2}}\,dx$

$dx = du \cdot \frac{\sqrt{1-x^2}}{-x}$

$= \int \frac{2x}{u} \cdot \frac{\sqrt{1-x^2}}{-x}\,du$

$= -2\int \frac{1}{u} \cdot u\,du$

$= -2\int_1^{\sqrt{3}/2} 1\,du$

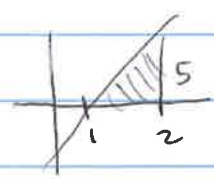
$= -2u \Big|_1^{\sqrt{3}/2} = -2\left(\frac{\sqrt{3}}{2} - 1\right) = -\sqrt{3} + 2$

(6)

(32) $\int \frac{5}{1+x^2}\,dx = 5\int \frac{1}{1+x^2}\,dx = 5\tan^{-1}x + C$

(7)

(38) $\int_1^2 f(x-c)\,dx = 5$



$\int_{1-c}^{2-c} f(x)\,dx = 5$

f(x) is the same function as f(x-c) except shifted c units to right.

Taking c away from limits make the same "area" under curve.

8

(4) $\frac{dy}{dx} = \cos(2x)$

$u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$= \int \cos(2x)\,dx$

$= \frac{1}{2}\int \cos u\,du$

$= \frac{1}{2}\sin u + C$

$= \boxed{\frac{1}{2}\sin 2x + C}$

9

(32) $\int_0^{\pi/3} \sin(3x)\,dx$

$u = 3x$

$du = 3dx$

$dx = \frac{du}{3}$

$\frac{1}{3}\int_0^{\pi} \sin u\,du$

$= -\frac{1}{3}\cos u \Big|_0^{\pi} = -\frac{1}{3}\left[\cos\pi - \cos 0\right]$

$= -\frac{1}{3}\left[-1 - 1\right]$

$= \boxed{2/3}$

(30) $\int \tan(2x)\,dx$

$u = 2x$

$du = 2\,dx$

$dx = \frac{du}{2}$

$= \frac{1}{2}\int \tan u \, du$

$= \frac{1}{2}\ln|\sec u| + c$

$= \frac{1}{2}\ln|\sec 2x| + c$

$= \frac{1}{2}\ln|(\cos 2x)|^{-1} + C$

$= -\frac{1}{2}\ln|\cos(2x)| + C$

(44) $f_{Ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$= \frac{1}{2}\int x^2\sqrt{u}\,\frac{du}{3x^2}$

$= \frac{1}{6}\int_1^9 u^{1/2}\,du$

$= \frac{1}{6}\cdot\frac{2u^{3/2}}{3}\Big|_1^9 = \frac{1}{9}u^{3/2}\Big|_1^9 = \frac{1}{9}\left[9^{3/2} - 1^{3/2}\right]$

$= \frac{1}{9}[27-1] = \frac{26}{9}$

(7) $\int \frac{x\,dx}{\sqrt{3x^2+5}}$

$u = 3x^2+5$

$du = 6x\,dx$

$dx = \frac{du}{6x}$

12

$\int \frac{x}{\sqrt{u}} \cdot \frac{du}{6x}$

$= \frac{1}{6}\int u^{-1/2}\,du = \frac{1}{6} \cdot \frac{u^{1/2}}{1/2} + C$

$= \frac{1}{3}\sqrt{u} + C$

$= \frac{1}{3}\sqrt{3x^2+5} + C$

M

(14) $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

13

$= \int \frac{3x^2}{\sqrt{u}} \frac{du}{3x^2}$

$= \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{x^3+1} + C$

14 (32) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{\left(\frac{4}{4}-\frac{x^2}{4}\right)4}} = \frac{1}{2}\int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$dx = 2\,du$

$= \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}} \cdot 2\,du$

$= \frac{1}{2} 2 \int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{2} 2 \sin^{-1} u \Big|_0^{\sqrt{3}/2}$

$= \frac{1}{2} 2 \left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}(0)\right]$

$= \frac{1}{2} 2 \left[\frac{\pi}{3} - 0\right] = \boxed{\frac{\pi}{3}}$

15

(6) $\frac{1}{2}\int e^{t/2}\,dt$

$u = \frac{1}{2}t$
$du = \frac{1}{2}dt$
$dt = 2du$

$= \frac{1}{2}\int e^{u}\, 2du$

$= \int e^{u}\, du$

$= e^{u} + c$

$= e^{1/2 t} + c$

16

(18) $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}\,dx$

$u = \tan x$
$du = \sec^2 x\, dx$
$dx = \frac{du}{\sec^2 x}$
$dx = du \cdot \cos^2 x$

$\int \frac{e^{u}}{\cos^2 x} \cos^2 x\, du$

$\int_0^1 e^{u}\, du$

$e^{u}\Big|_0^1 = e - e^0 = e - 1$

17 (27) $f_{ave} = \frac{1}{2-0}\int x^2\sqrt{x^3+1}$

Same prob as 44

18 (82) $F'(x) = f(x)$

$$\int_1^3 f(2x)\,dx$$

$u = 2x$
$du = 2dx$

$$\int f(u)\,\frac{du}{2} = \frac{1}{2}\int_2^6 f(u)\,du$$

$$= \frac{1}{2}\left[F(6) - F(2)\right]$$

19 (14) $$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta$$

$u = 1+\sin\theta$
$du = \cos\theta\,d\theta$
$d\theta = \frac{du}{\cos\theta}$

$$\int_1^2 \frac{\cos\theta}{\sqrt{u}}\,\frac{du}{\cos\theta} = \int_1^2 u^{-1/2}\,du$$

$$2u^{1/2}\Big|_1^2 = 2\sqrt{2} - 2$$

$$= 2(\sqrt{2}-1)$$

(42) $\int_1^4 f(x)\,dx = 6$

20

$\int_1^4 f(5-x)\,dx = \int_4^1 f(u)\,(-du) = -\int_4^1 f(u)\,du$

$= \int_1^4 f(u)\,du = \underline{\underline{6}}$

let $u = 5-x$

$du = -dx$

$dx = -du$

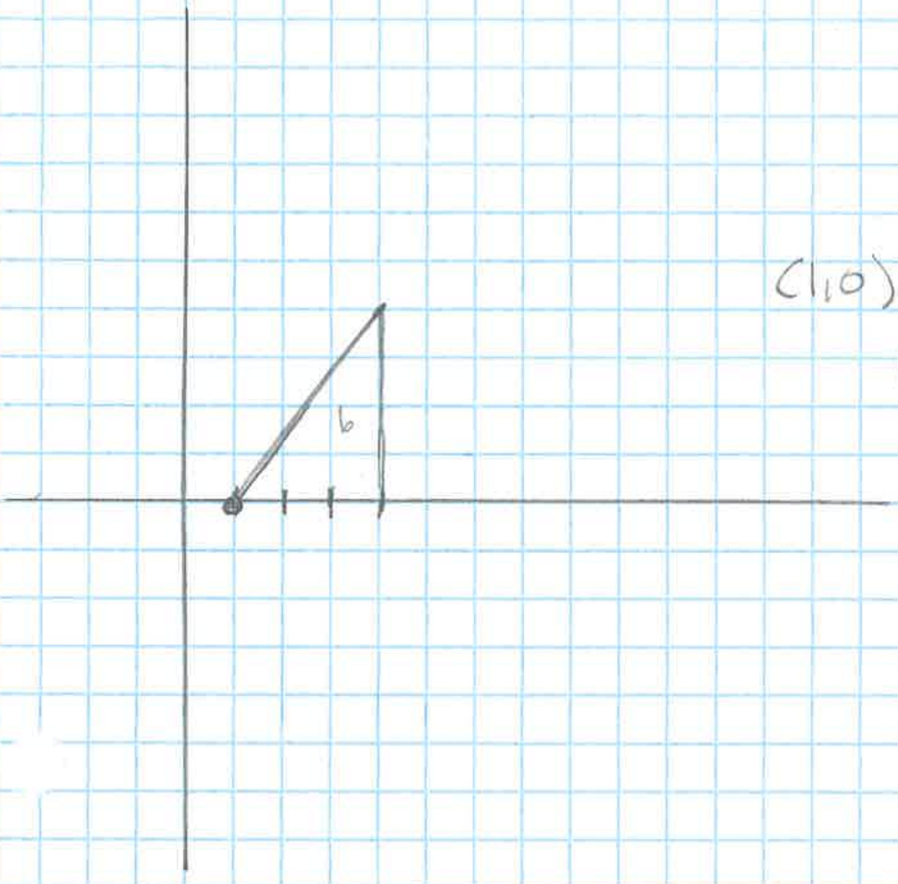
$x=1$: $u = 5-1 = 4$

$x=4$: $u = 5-4 = 1$

(20)

(42)

Geometric interpretation of 42



(1,0) (4,4)

$m = \frac{4}{3}$

$y - 0 = \frac{4}{3}(x-1)$

$y = \frac{4}{3}x - \frac{4}{3}$

$f(x) = \frac{4}{3}x - \frac{4}{3}$

$f(5-x) = \frac{4}{3}(5-x) - \frac{4}{3}$

$\frac{20}{3} - \frac{4}{3}x - \frac{4}{3}$

$\frac{16}{3} - \frac{4}{3}x$

15

$\int_1^4 \frac{16}{3} - \frac{4}{3}x$

$\frac{16}{3}x - \frac{4}{3}\frac{1}{2}x^2$

$\frac{16}{3}x - \frac{2}{3}x^2 \Big|_1^4$

$\left(\frac{16}{3}(4) - \frac{2}{3} \cdot 16\right) - \left(\frac{16}{3} - \frac{2}{3}\right)$

$10^{2}/_{3} - {}^{14}/_{3} = (6)$

$\sqrt{0} = \sin y$ $\sqrt{\tfrac{1}{2}} = \sin y$
$y = 0$ $y = \pi/4$

$\sqrt{x} = \sin y$ $dx = 2\sin y \cdot \cos y \, dy$
$x = \sin^2 y$

22

(28) $\int \frac{\sin y}{\sqrt{1-\sin^2 y}} dx = \int \frac{\sin y}{\sqrt{\cos^2 y}} dx = \int_0^{\pi/4} \frac{\sin y}{\cos y} \cdot 2\sin y \cos y$

$= \int 2\sin^2 y \, dy$

$= 2\int_0^{\pi/4} \sin^2 y \, dy$

23

(3) $\int_1^2 \frac{x+1}{x^2+2x} dx$

$u = x^2+2x$
$du = 2x+2 \, dx$
$dx = \frac{du}{2(x+1)}$
$x=1: u = 1^2+2(1) = 3$
$x=2 \quad u = 2^2+2\cdot2 = 8$

$\int_3^8 \frac{x+1}{u} \cdot \frac{du}{2(x+1)}$

$\frac{1}{2}\int_3^8 \frac{1}{u} du$

$\frac{1}{2} \ln|u| \Big|_3^8 = \frac{1}{2}[\ln 8 - \ln 3] = \frac{1}{2}\ln 8/3$ (B)

24

(45) $u = \frac{x}{2}$
$x = 2u$

$\int_1^2 \frac{1-u^2}{2u} du$ (A)

$x=2 = u = \frac{2}{2} = 1$
$x=4 = u = \frac{4}{2} = 2$

(20) $\int x\sqrt{4-x^2}\,dx$

$u = 4-x^2$
$du = -2x\,dx$
$dx = \frac{du}{-2x}$

21

$\int x\sqrt{u}\,\frac{du}{-2x}$

$-\frac{1}{2}\int u^{1/2}\,du$

$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\frac{-1}{3}u^{3/2} + C$

$= -\frac{1}{3}(4-x^2)^{3/2} + C$

$= \frac{-(4-x^2)^{3/2}}{3} + C$

25 (7) $\ln 4 =$

$$\int_1^4 \frac{1}{t}\,dt = \ln|t| \Big|_1^4 = \ln 4 - \ln 1$$
$$= \ln 4 \quad \text{(E)}$$

26 (7) $\int \frac{1}{\sqrt{25-x^2}}\,dx = \frac{1}{5}\int \frac{1}{\sqrt{1-\left(\frac{x}{5}\right)^2}}\,dx$

$u = \frac{x}{5}$

$du = \frac{1}{5}dx$

$dx = 5\,du$

$$\frac{1}{5}\cdot 5\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C \quad \text{(A)}$$
$$= \sin^{-1}\left(\frac{x}{5}\right) + C$$

27 (21) $F_{ave} = \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx$

$$\frac{1}{2}\ln|x| \Big|_1^3$$

(D) $\frac{1}{2}\left[\ln 3 - \ln 1\right]$

$= \frac{1}{2}\ln 3$

25 (-2) ln 4 =

$\int_1^4 \frac{1}{t}\,dt = \ln|t|\Big|_1^4 = \ln 4 - \ln 1$

$= \ln 4$ (E)

26 (1) $\int \frac{1}{\sqrt{25-x^2}}\,dx = \frac{1}{5}\int \frac{1}{\sqrt{1-(\frac{x}{5})^2}}\,dx$

$u = \frac{x}{5}$

$du = \frac{1}{5}dx$

$dx = 5\,du$

$\frac{1}{5}\cdot 5 \int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C$

$= \sin^{-1}\left(\frac{x}{5}\right) + C$

(A)

27 (2) $f_{ave} = \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx$

$\frac{1}{2}\ln|x|\Big|_1^3$

$\frac{1}{2}[\ln 3 - \ln 1]$

$\frac{1}{2}\ln 3$

(D)