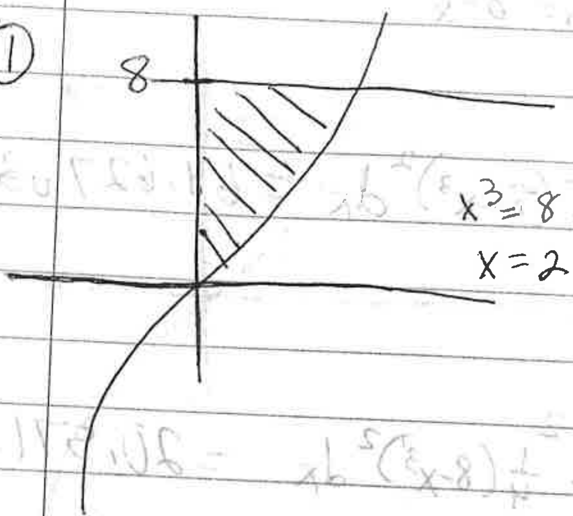
AP Calculus AB Homework Problems

1) Find the volume of the solid formed by the functions $y = x^3$ **and** $y = 8$ **and** $x = 0$ with cross sectional shapes perpendicular to the x-axis:

   a) Square with side perpendicular
   b) Square with diagonal perpendicular
   c) Semi-Circles
   d) Circle
   e) **45°- 45°- 90°** Triangles with hypotenuse perpendicular
   f) **30°- 60°- 90°** Triangle's with shortest side perpendicular

2) Now re-do the problems above when the cross sectional shapes are perpendicular to the y-axis.

X-axis

① 

$S = 8 - x^3$

$x^3 = 8$

$x = 2$

ⓐ Square w/ side ⊥ x-axis

$A = s^2$

$A = (8-x^3)^2$ $\qquad V = \int_0^2 (8-x^3)^2\,dx = 82.286\,u^3$

ⓑ square w/ diag ⊥

$\frac{s}{\sqrt{2}}$ $\qquad A = \left(\frac{s}{\sqrt{2}}\right)^2 = \frac{s^2}{2}$

$$V = \int_0^2 \frac{s^2}{2}\,dx = \int_0^2 \frac{1}{2}(8-x^3)^2\,dx = 41.143\,u^3$$

ⓒ Semi-circle

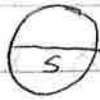
$r = s/2 \qquad A = \frac{1}{8}\pi s^2$

$A = \frac{1}{2}\pi r^2$

$d = s \qquad \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 \qquad V = \int_0^2 \frac{\pi}{8}(8-x^3)^2\,dx = 32.314\,u^3$

$S = 8 - x^3$

d) circle



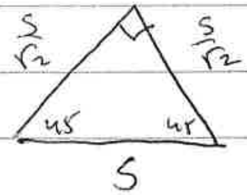
$A = \pi r^2$

$A = \pi \left(\frac{s}{2}\right)^2$

$A = \frac{\pi}{4} s^2$

$$V = \int_0^2 \frac{\pi}{4}(8-x^3)^2\,dx = 64.627\ u^3$$

e)

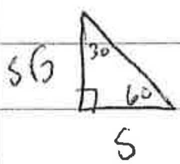


$$V = \int_0^2 \frac{1}{4}(8-x^3)^2\,dx = 20.571\ u^3$$

$A = \frac{1}{2}\left(\frac{s}{\sqrt{2}}\right)^2$

$A = \frac{1}{4}s^2$

f)

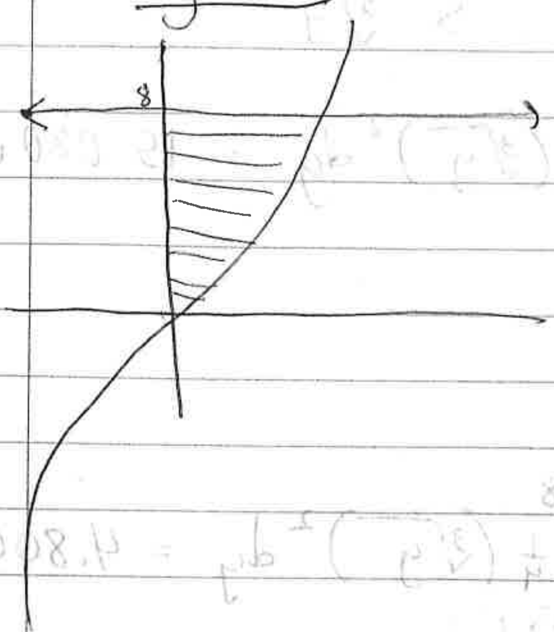


$$V = \int_0^2 \frac{\sqrt{3}}{2}(8-x^3)^2 = 71.262\ u^3$$

$A = \frac{1}{2} s \cdot s\sqrt{3}$

$= \frac{\sqrt{3}}{2} s^2$

y-axis



$y = x^3$

$x = \sqrt[3]{y}$

$\int_0^8 A(x)$ $\quad s = \sqrt[3]{y} - 0$

(a) Square w/ side ⊥ y-axis

$A = s^2$

$V = \int_0^8 (\sqrt[3]{y})^2 \, dy = 19.200 \, u^3$

(b) Squre w/ diag. ⊥

s, $\frac{s}{\sqrt{2}}$, $\frac{s}{\sqrt{2}}$

$V = \int_0^8 \frac{1}{2}(\sqrt[3]{y})^2 \, dy = 9.600 \, u^3$

$A = s^2$

$A = \left(\frac{s}{\sqrt{2}}\right)^2 = \frac{1s^2}{2}$

(c) Semicircle

$V = \int_0^8 \frac{\pi}{8}(\sqrt[3]{y})^2 \, dy = 7.540 \, u^3$

s

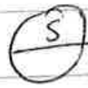
$A = \frac{1}{2}\pi r^2$

$\frac{1}{2}\pi\left(\frac{s}{2}\right)^2$

$d = s$

$r = 1/2 s$ $\quad \frac{1}{8}\pi s^2$

$S = \sqrt[3]{y}$

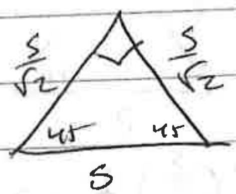
(d) circle 

$$V = \int_0^8 \frac{\pi}{4}\left(\sqrt[3]{y}\right)^2 dy = 15.080\,u^3$$

$A = \pi r^2$

$A = \pi\left(\frac{s}{2}\right)^2$

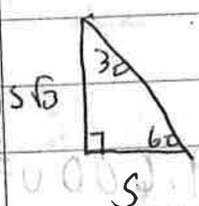
$A = \frac{\pi}{4}s^2$

(e) 

$$V = \int_0^8 \frac{1}{4}\left(\sqrt[3]{y}\right)^2 dy = 4.800\,u^3$$

$A = \frac{1}{2}\left(\frac{s}{\sqrt{2}}\right)^2$

$= \frac{1}{4}s^2$

(f) 

$$V = \int_0^8 \frac{\sqrt{3}}{2}\left(\sqrt[3]{y}\right)^2 dy = 16.628\,u^3$$

$A = \frac{1}{2}\, s \cdot s\sqrt{3}$

$A = \frac{\sqrt{3}}{2}s^2$