Honors Pre-Calculus Integral/Riemann Sum Formative Quiz

1. $\int_{-1}^{3} x|x-2|dx$

$$\begin{cases} -(x-2) & x<2 \\ (x-2) & x\geq 2 \end{cases}$$

$$\begin{cases} -x(x-2) & x<2 \\ x(x-2) & x\geq 2 \end{cases}$$

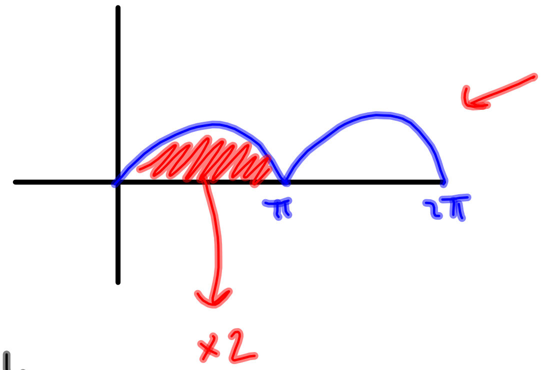
$$\begin{cases} -x^2+2x & x<2 \\ x^2-2x & x\geq 2 \end{cases}$$

$$\int_{-1}^{2} (-x^2+2x)dx + \int_{2}^{3} (x^2-2x)dx$$

$$-\frac{x^3}{3} + x^2 \Big|_{-1}^{2} + \frac{x^3}{3} - x^2 \Big|_{2}^{3}$$

$$\boxed{\frac{4}{3}}$$

2. $\int_0^{2\pi} |\sin x|\, dx$



$2 \cdot \int_0^{\pi} \sin x \, dx$

$2 \left(-\cos x\right)\Big|_0^{\pi}$

$2\left[-\cos(\pi) - (-\cos 0)\right]$

$2[1 + 1]$

$\boxed{4}$

3. Given $F(x)=\int_1^x f(t)dt$ and $f(t)=\int_t^{t^2}\left(u-\frac{1}{u}\right)du$, find $F''(2)$

i) $F'(x)=f(x)$

(ii) $f(x)=\int_x^{x^2}\left(u-\frac{1}{u}\right)du$

(iii) $F''(x)=f'(x)=\left(x^2-\frac{1}{x^2}\right)2x-\left(x-\frac{1}{x}\right)$

$=2x^3-\frac{2}{x}-x+\frac{1}{x}$

$=2x^3-\frac{1}{x}-x$

IV) $F''(2)=2(2)^3-\frac{1}{2}-2$

$=16-\frac{1}{2}-2$

$=14-\frac{1}{2}=\boxed{13\frac{1}{2}}$

4. The average value of $2x-1$ on $[3,a]$ is 9. Find the value of a.

$$\frac{1}{a-3}\int_3^a (2x-1)\,dx = 9$$

$$\int_3^a (2x-1)\,dx = 9a-27$$

$$x^2 - x \Big|_3^a = 9a-27$$

$$(a^2-a)-(9-3) = 9a-27$$

$$a^2-a-6 = 9a-27$$

$$a^2-10a+21 = 0$$

$$(a-7)(a-3)$$

$$a = 7, 3$$

$$\boxed{a = 7 \text{ only!}}$$

5. Given $g(x) = x^2 + \int_2^x (t-1)\,dt$, find the tangent line to g at $x = 3$.

i) Point

ii) Slope

$$g(3) = 9 + \int_2^3 (t-1)\,dt$$

$$= 9 + \left.\frac{t^2}{2} - t\right|_2^3$$

$$= 9 + \left(\frac{9}{2} - 3\right) - (2-2)$$

$$= 6 + \frac{9}{2}$$

$$= \frac{21}{2}$$

Point $\left(3, \frac{21}{2}\right)$

$$g'(x) = 2x + x - 1$$

$$g'(x) = 3x - 1$$

$$g'(3) = 9 - 1$$

$$= 8$$

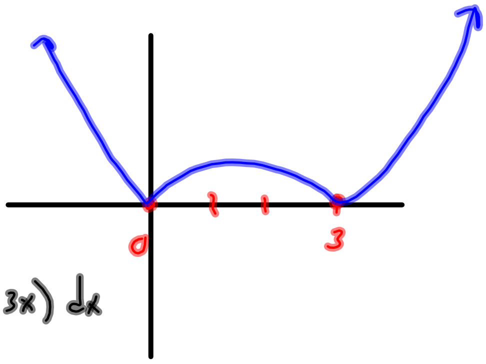
$$m = 8$$

$$\boxed{y - \frac{21}{2} = 8(x-3)}$$

6. $\int_{-2}^{3} |x^2 - 3x| \, dx = \frac{79}{6}$

$x(x-3)$

$x=0 \quad x=3$



$\int_{-2}^{0} (x^2-3x)\,dx - \int_{0}^{3} (x^2-3x)\,dx$

7. $\int_{-1}^{1} \frac{4}{1+x^2}\,dx$

P. 406

$$4\tan^{-1}x \Big|_{-1}^{1}$$

$$4\left[\tan^{-1}1 - \tan^{-1}-1\right]$$

$$4\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right]$$

$$4\left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$\boxed{2\pi}$$

8. Write the integral that correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$.

$-(x^2 - 2x - 8)$

$-(x-4)(x+2)$

$x = 4, -2$

$$\int_{-2}^{4} (8 + 2x - x^2)\,dx$$

9. $\int_2^{x^2} \sin(t)dt$

$$-\cos t \Big|_2^{x^2} = -[\cos x^2 - \cos 2]$$

or

$$\boxed{-\cos x^2 + \cos 2}$$

10. $\dfrac{d}{dx}\displaystyle\int_{2x}^{5x} \cos(t)\,dt$

$5\cos(5x) - 2\cos(2x)$

11. Find the average value of $\sec^2 x$ on $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$.

$$\frac{1}{\frac{\pi}{4} - \frac{\pi}{6}} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$\frac{1}{\frac{3\pi}{12} - \frac{2\pi}{12}} \left[\tan x \right] \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$\frac{12}{\pi} \left[\tan \frac{\pi}{4} - \tan \frac{\pi}{6} \right]$$

$$\boxed{\frac{12}{\pi} \left[1 - \frac{\sqrt{3}}{3} \right]}$$

12. If $\int_{-2}^{4} f(x)dx = a$ and $\int_{3}^{4} f(x)dx = b$, then $\int_{3}^{-2} f(x)dx =$

$$\int_{-2}^{4} = \int_{-2}^{3} + \int_{3}^{4}$$

x

$$a = x + b$$

$$a - b = x$$

$$a - b = \int_{-2}^{3}$$ so

$$-(a-b) = \int_{3}^{-2}$$

$-a+b$ or $\rightarrow$ $\boxed{b-a}$