Set 5: Multiple-Choice Questions on Integration

1. $\int (3x^2 - 2x + 3)\, dx =$

(A) $x^3 - x^2 + C$ **(B)** $3x^3 - x^2 + 3x + C$ **(C)** $x^3 - x^2 + 3x + C$

(D) $\frac{1}{2}(3x^2 - 2x + 3)^2 + C$ **(E)** none of these

2. $\int \left(x - \frac{1}{2x}\right)^2 dx =$

(A) $\frac{1}{3}\left(x - \frac{1}{2x}\right)^3 + C$ **(B)** $x^2 - 1 + \frac{1}{4x^2} + C$ **(C)** $\frac{x^3}{3} - 2x - \frac{1}{4x} + C$

(D) $\frac{x^3}{3} - x - \frac{4}{x} + C$ **(E)** none of these

3. $\int \sqrt{4 - 2t}\; dt =$

(A) $-\frac{1}{3}(4 - 2t)^{3/2} + C$ **(B)** $\frac{2}{3}(4 - 2t)^{3/2} + C$ **(C)** $-\frac{1}{6}(4 - 2t)^3 + C$

(D) $+\frac{1}{2}(4 - 2t)^2 + C$ **(E)** $\frac{4}{3}(4 - 2t)^{3/2} + C$

4. $\int (2 - 3x)^5\, dx =$

(A) $\frac{1}{6}(2 - 3x)^6 + C$ **(B)** $-\frac{1}{2}(2 - 3x)^6 + C$ **(C)** $\frac{1}{2}(2 - 3x)^6 + C$

(D) $-\frac{1}{18}(2 - 3x)^6 + C$ **(E)** none of these

5. $\int \frac{1 - 3y}{\sqrt{2y - 3y^2}}\, dy =$

(A) $4\sqrt{2y - 3y^2} + C$ **(B)** $\frac{1}{4}(2y - 3y^2)^2 + C$ **(C)** $\frac{1}{2}\ln\sqrt{2y - 3y^2} + C$

(D) $\frac{1}{4}(2y - 3y^2)^{1/2} + C$ **(E)** $\sqrt{2y - 3y^2} + C$

6. $\int \frac{dx}{3(2x - 1)^2} =$

(A) $\frac{-3}{2x - 1} + C$ **(B)** $\frac{1}{6 - 12x} + C$ **(C)** $+\frac{6}{2x - 1} + C$

(D) $\frac{2}{3\sqrt{2x - 1}} + C$ **(E)** $\frac{1}{3}\ln|2x - 1| + C$

7. $\int \frac{2\,du}{1 + 3u} =$

(A) $\frac{2}{3}\ln|1 + 3u| + C$ **(B)** $-\frac{1}{3(1 + 3u)^2} + C$ **(C)** $2\ln|1 + 3u| + C$

(D) $\frac{3}{(1 + 3u)^2} + C$ **(E)** none of these

8. $\int \frac{t}{\sqrt{2t^2 - 1}}\, dt =$

(A) $\frac{1}{2}\ln\sqrt{2t^2 - 1} + C$ **(B)** $4\ln\sqrt{2t^2 - 1} + C$ **(C)** $8\sqrt{2t^2 - 1} + C$

(D) $-\frac{1}{4(2t^2 - 1)} + C$ **(E)** $\frac{1}{2}\sqrt{2t^2 - 1} + C$

9. $\int \cos 3x\, dx =$

(A) $3\sin 3x + C$ **(B)** $-\sin 3x + C$ **(C)** $-\frac{1}{3}\sin 3x + C$

(D) $\frac{1}{3}\sin 3x + C$ **(E)** $\frac{1}{2}\cos^2 3x + C$

10. $\int \frac{x\,dx}{1 + 4x^2} =$

(A) $\frac{1}{8}\ln(1 + 4x^2) + C$ **(B)** $\frac{1}{8(1 + 4x^2)^2} + C$ **(C)** $\frac{1}{4}\sqrt{1 + 4x^2} + C$

(D) $\frac{1}{2}\ln\left|1 + 4x^2\right| + C$ **(E)** $\frac{1}{2}\tan^{-1} 2x + C$

11. $\int \frac{dx}{1 + 4x^2} =$

(A) $\tan^{-1}(2x) + C$ **(B)** $\frac{1}{8}\ln(1 + 4x^2) + C$ **(C)** $\frac{1}{8(1 + 4x^2)^2} + C$

(D) $\frac{1}{2}\tan^{-1}(2x) + C$ **(E)** $\frac{1}{8x}\ln\left|1 + 4x^2\right| + C$

12. $\int \frac{x}{(1+4x^2)^2}\,dx =$

(A) $\frac{1}{8}\ln(1+4x^2)^2 + C$ **(B)** $\frac{1}{4}\sqrt{1+4x^2} + C$ **(C)** $-\frac{1}{8(1+4x^2)} + C$

(D) $-\frac{1}{3(1+4x^2)^3} + C$ **(E)** $-\frac{1}{(1+4x^2)} + C$

13. $\int \frac{x\,dx}{\sqrt{1+4x^2}} =$

(A) $\frac{1}{8}\sqrt{1+4x^2} + C$ **(B)** $\frac{\sqrt{1+4x^2}}{4} + C$ **(C)** $\frac{1}{2}\sin^{-1}2x + C$

(D) $\frac{1}{2}\tan^{-1}2x + C$ **(E)** $\frac{1}{8}\ln\sqrt{1+4x^2} + C$

14. $\int \frac{dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ **(B)** $-\sqrt{4-y^2} + C$ **(C)** $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ **(E)** $-\frac{1}{3(4-y^2)^{3/2}} + C$

15. $\int \frac{y\,dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ **(B)** $-\sqrt{4-y^2} + C$ **(C)** $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ **(E)** $2\sqrt{4-y^2} + C$

16. $\int \frac{2x+1}{2x}\,dx =$

(A) $x + \frac{1}{2}\ln|x| + C$ **(B)** $1 + \frac{1}{2}x^{-1} + C$ **(C)** $x + 2\ln|x| + C$

(D) $x + \ln|2x| + C$ **(E)** $\frac{1}{2}\left(2x - \frac{1}{x^2}\right) + C$

17. $\int \frac{(x-2)^3}{x^2}\,dx =$

(A) $\frac{(x-2)^4}{4x^2} + C$ **(B)** $\frac{x^2}{2} - 6x + 6\ln|x| - \frac{8}{x} + C$

(C) $\frac{x^2}{2} - 3x + 6\ln|x| + \frac{4}{x} + C$ **(D)** $-\frac{(x-2)^4}{4x} + C$

(E) none of these

18. $\int \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 dt =$

(A) $t - 2 + \frac{1}{t} + C$ **(B)** $\frac{t^3}{3} - 2t - \frac{1}{t} + C$ **(C)** $\frac{t^2}{2} + \ln|t| + C$

(D) $\frac{t^2}{2} - 2t + \ln|t| + C$ **(E)** $\frac{t^2}{2} - t - \frac{1}{t^2} + C$

19. $\int (4x^{1/3} - 5x^{3/2} - x^{-1/2})\,dx =$

(A) $3x^{4/3} - 2x^{5/2} - 2x^{1/2} + C$

(B) $3x^{4/3} - 2x^{5/2} + 2x^{1/2} + C$

(C) $6x^{2/3} - 2x^{5/2} - \frac{1}{2}x^2 + C$

(D) $\frac{4}{3}x^{-2/3} - \frac{15}{2}x^{1/2} + \frac{1}{2}x^{-3/2} + C$

(E) none of these

20. $\int \frac{x^3 - x - 1}{(x+1)^2}\,dx =$

(A) $(x-2) + \frac{2x+1}{(x+1)^2} + C$

(B) $x^2 - 2x + \frac{1}{2}\ln(x^2 + 2x + 1) + C$

(C) $\frac{1}{2}x^2 - 2x + \ln|x+1|^2 - \frac{1}{x+1} + C$

(D) $\frac{1}{2}(x-2)^2 + 2\ln|x+1| + \frac{1}{x+1} + C$

(E) none of these

21. $\int \frac{dy}{\sqrt{y}(1-\sqrt{y})} =$

(A) $4\sqrt{1-\sqrt{y}} + C$ **(B)** $\frac{1}{2}\ln\left|1-\sqrt{y}\right| + C$ **(C)** $2\ln(1-\sqrt{y}) + C$

(D) $2\sqrt{y} - \ln|y| + C$ **(E)** $-2\ln\left|1-\sqrt{y}\right| + C$

22. $\int \frac{u\,du}{\sqrt{4-9u^2}} =$

(A) $\frac{1}{3}\sin^{-1}\frac{3u}{2} + C$ **(B)** $-\frac{1}{18}\ln\sqrt{4-9u^2} + C$ **(C)** $2\sqrt{4-9u^2} + C$

(D) $\frac{1}{6}\sin^{-1}\frac{3}{2}u + C$ **(E)** $-\frac{1}{9}\sqrt{4-9u^2} + C$

23. $\int \sin\theta\cos\theta\, d\theta =$

(A) $-\frac{\sin^2\theta}{2} + C$ (B) $-\frac{1}{4}\cos 2\theta + C$ (C) $\frac{\cos^2\theta}{2} + C$

(D) $\frac{1}{2}\sin 2\theta + C$ (E) $\cos 2\theta + C$

24. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx =$

(A) $-2\cos^{1/2} x + C$ (B) $-\cos\sqrt{x} + C$ (C) $-2\cos\sqrt{x} + C$

(D) $\frac{3}{2}\sin^{3/2} x + C$ (E) $\frac{1}{2}\cos\sqrt{x} + C$

25. $\int t\cos(2t)^2\, dt =$

(A) $\frac{1}{8}\sin(4t^2) + C$ (B) $\frac{1}{2}\cos^2(2t) + C$ (C) $-\frac{1}{8}\sin(4t^2) + C$

(D) $\frac{1}{4}\sin(2t)^2 + C$ (E) none of these

27. $\int \sin 2\theta\, d\theta =$

(A) $\frac{1}{2}\cos 2\theta + C$ (B) $-2\cos 2\theta + C$ (C) $-\sin^2\theta + C$

(D) $\cos^2\theta + C$ (E) $-\frac{1}{2}\cos 2\theta + C$

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30. $\int \frac{\cos x\, dx}{\sqrt{1+\sin x}} =$

(A) $-\frac{1}{2}(1+\sin x)^{1/2} + C$

(B) $\ln\sqrt{1+\sin x} + C$

(C) $2\sqrt{1+\sin x} + C$

(D) $\ln|1+\sin x| + C$

(E) $\frac{2}{3(1+\sin x)^{3/2}} + C$

33. $\int \frac{\sin 2x\, dx}{\sqrt{1+\cos^2 x}} =$

(A) $-2\sqrt{1+\cos^2 x} + C$ (B) $\frac{1}{2}\ln(1+\cos^2 x) + C$

(C) $\sqrt{1+\cos^2 x} + C$ (D) $-\ln\sqrt{1+\cos^2 x} + C$

(E) $2\ln|\sin x| + C$

35. $\int \tan\theta\, d\theta =$

(A) $-\ln|\sec\theta| + C$ (B) $\sec^2\theta + C$ (C) $\ln|\sin\theta| + C$

(D) $\sec\theta + C$ (E) $-\ln|\cos\theta| + C$

37. $\int \frac{\tan^{-1} y}{1 + y^2}\, dy =$

(A) $\sec^{-1} y + C$ **(B)** $(\tan^{-1} y)^2 + C$ **(C)** $\ln(1 + y^2) + C$
(D) $\ln(\tan^{-1} y) + C$ **(E)** none of these

39. $\int \frac{\sin 2t}{1 - \cos 2t}\, dt =$

(A) $\frac{2}{(1 - \cos 2t)^2} + C$ **(B)** $-\ln|1 - \cos 2t| + C$ **(C)** $\ln\sqrt{|1 - \cos 2t|} + C$

(D) $\sqrt{1 - \cos 2t} + C$ **(E)** $2\ln|1 - \cos 2t| + C$

40. $\int \cot 2u\, du =$

(A) $\ln|\sin u| + C$ **(B)** $\frac{1}{2}\ln|\sin 2u| + C$ **(C)** $-\frac{1}{2}\csc^2 2u + C$

(D) $-\sec 2u + C$ **(E)** $2\ln|\sin 2u| + C$

41. $\int \frac{e^x}{e^x - 1}\, dx =$

(A) $x + \ln|e^x - 1| + C$ **(B)** $x - e^x + C$ **(C)** $x - \frac{1}{(e^x - 1)^2} + C$

(D) $1 + \frac{1}{e^x - 1} + C$ **(E)** $\ln|e^x - 1| + C$

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43. $\int xe^{x^2}\, dx =$

(A) $\frac{1}{2}e^{x^2} + C$ **(B)** $e^{x^2}(2x^2 + 1) + C$ **(C)** $2e^{x^2} + C$

(D) $e^{x^2} + C$ **(E)** $\frac{1}{2}e^{x^2+1} + C$

44. $\int \cos\theta\, e^{\sin\theta}\, d\theta =$

(A) $e^{\sin\theta + 1} + C$ **(B)** $e^{\sin\theta} + C$ **(C)** $-e^{\sin\theta} + C$

(D) $e^{\cos\theta} + C$ **(E)** $e^{\sin\theta}(\cos\theta - \sin\theta) + C$

45. $\int e^{2\theta} \sin e^{2\theta}\, d\theta =$

(A) $\cos e^{2\theta} + C$ **(B)** $2e^{4\theta}(\cos e^{2\theta} + \sin e^{2\theta}) + C$ **(C)** $-\frac{1}{2}\cos e^{2\theta} + C$

(D) $-2\cos e^{2\theta} + C$ **(E)** none of these

46. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx =$

(A) $2\sqrt{x}(e^{\sqrt{x}} - 1) + C$ **(B)** $2e^{\sqrt{x}} + C$ **(C)** $\frac{e^{\sqrt{x}}}{2}\left(\frac{1}{x} + \frac{1}{x\sqrt{x}}\right) + C$

(D) $\frac{1}{2}e^{\sqrt{x}} + C$ **(E)** none of these

*An asterisk denotes a topic covered only in Calculus BC.

50. $\int \frac{e^x}{1+e^{2x}}\,dx =$

(A) $\tan^{-1} e^x + C$ **(B)** $\frac{1}{2}\ln(1+e^{2x}) + C$ **(C)** $\ln(1+e^{2x}) + C$

(D) $\frac{1}{2}\tan^{-1} e^x + C$ **(E)** $2\tan^{-1} e^x + C$

51. $\int \frac{\ln v\,dv}{v} =$

(A) $\ln|\ln v| + C$ **(B)** $\ln\frac{v^2}{2} + C$ **(C)** $\frac{1}{2}(\ln v)^2 + C$

(D) $2\ln v + C$ **(E)** $\frac{1}{2}\ln v^2 + C$

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57. $\int \frac{dv}{v\ln v} =$

(A) $\frac{1}{\ln v^2} + C$ **(B)** $-\frac{1}{\ln^2 v} + C$ **(C)** $-\ln|\ln v| + C$

(D) $\ln\frac{1}{v} + C$ **(E)** $\ln|\ln v| + C$

58. $\int \frac{y-1}{y+1}\,dy =$

(A) $y - 2\ln|y+1| + C$ **(B)** $1 - \frac{2}{y+1} + C$ **(C)** $\ln\frac{|y|}{(y+1)^2} + C$

(D) $1 - 2\ln|y+1| + C$ **(E)** $\ln\left|\frac{e^y}{y+1}\right| + C$

59. $\int t\sqrt{t+1}\,dt =$

(A) $\frac{2}{3}(t+1)^{3/2} + C$ **(B)** $\frac{2}{15}(3t-2)(t+1)^{3/2} + C$

(C) $2\left[\frac{(t+1)^{5/2}}{5} + \frac{(t+1)^{3/2}}{5}\right] + C$ **(D)** $2t(t+1) + C$

(E) none of these

60. $\int \sqrt{x}(\sqrt{x}-1)\,dx =$

(A) $2(x^{3/2} - x) + C$ **(B)** $\frac{x^2}{2} - x + C$ **(C)** $\frac{1}{2}(\sqrt{x}-1)^2 + C$

(D) $\frac{1}{2}x^2 - \frac{2}{3}x^{3/2} + C$ **(E)** $x - 2\sqrt{x} + C$

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62. $\int \frac{(1-\ln t)^2}{t}\, dt =$

(A) $\frac{1}{3}(1-\ln t)^3 + C$ (B) $\ln t - 2\ln^2 t + \ln^3 t + C$ (C) $-2(1-\ln t) + C$

(D) $\ln t - \ln^2 t + \frac{\ln t^3}{3} + C$ (E) $-\frac{(1-\ln t)^3}{3} + C$

64. $\int \frac{2x+1}{4+x^2}\, dx =$

(A) $\ln(x^2+4) + C$ (B) $\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$ (C) $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

(D) $\ln(x^2+4) + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$ (E) none of these

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68. $\int \frac{\cos\theta}{1+\sin^2\theta}\, d\theta =$

(A) $\sec\theta\tan\theta + C$ (B) $\sin\theta - \csc\theta + C$ (C) $\ln(1+\sin^2\theta) + C$

(D) $\tan^{-1}(\sin\theta) + C$ (E) $-\frac{1}{(1+\sin^2\theta)^2} + C$

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Answers for Set 5: Integration

1.	C	**17.**	E	**33.**	A	**49.**	D	**65.**	C
2.	E	**18.**	D	**34.**	D	**50.**	A	**66.**	E
3.	A	**19.**	A	**35.**	E	**51.**	C	**67.**	B
4.	D	**20.**	D	**36.**	C	**52.**	E	**68.**	D
5.	E	**21.**	E	**37.**	E	**53.**	B	**69.**	E
6.	B	**22.**	E	**38.**	A	**54.**	B	**70.**	B
7.	A	**23.**	B	**39.**	C	**55.**	B	**71.**	D
8.	E	**24.**	C	**40.**	B	**56.**	D	**72.**	C
9.	D	**25.**	A	**41.**	E	**57.**	E	**73.**	A
10.	A	**26.**	A	**42.**	D	**58.**	A	**74.**	B
11.	D	**27.**	E	**43.**	A	**59.**	B	**75.**	E
12.	C	**28.**	B	**44.**	B	**60.**	D	**76.**	D
13.	B	**29.**	D	**45.**	C	**61.**	C	**77.**	D
14.	C	**30.**	C	**46.**	B	**62.**	E	**78.**	D
15.	B	**31.**	B	**47.**	C	**63.**	A	**79.**	B
16.	A	**32.**	E	**48.**	C	**64.**	D	**80.**	A

All the references in parentheses below are to the basic integration formulas on pages 149 and 150. In general, if u is a function of x, then $du = u'(x)\,dx$.

1. C. Use, first, formula (2), then (3), replacing u by x.

2. E. Hint: Expand. The correct answer is $\frac{x^3}{3} - x - \frac{1}{4x} + C$.

3. A. By formula (3), with $u = 4 - 2t$ and $n = \frac{1}{2}$,

$$\int \sqrt{4-2t}\,dx = -\frac{1}{2}\int \sqrt{4-2t}\cdot(-2)\,dt = -\frac{1}{2}\frac{(4-2t)^{3/2}}{3/2} + C.$$

4. D. Use (3) with $u = 2 - 3x$, noting that $du = -3\,dx$.

5. E. Rewrite:

$$\int (2y-3y^2)^{-1/2}(1-3y)\,dy = \frac{1}{2}\int (2y-3y^2)^{-1/2}(2-6y)\,dy.$$

Use (3).

6. B. Rewrite:

$$\frac{1}{3}\int (2x-1)^{-2}\,dx = \frac{1}{3}\cdot\frac{1}{2}\int (2x-1)^{-2}\cdot 2dx.$$

Using (3) yields $-\frac{1}{6(2x-1)} + C$.

7. A. This is equivalent to $\frac{2}{3}\int \frac{dv}{v}$. Use (4).

8. E. Rewrite and modify to $\frac{1}{4}\int (2t^2 - 1)^{-1/2} \cdot 4t\,dt$. Use (3).

9. D. Use (5) with $u = 3x$; $du = 3\,dx$.

10. A. Use (4). If $u = 1 + 4x^2$, $du = 8x\,dx$.

11. D. Use (18). Let $u = 2x$; then $du = 2\,dx$.

12. C. Rewrite and modify to $\frac{1}{8}\int (1 + 4x^2)^{-2} \cdot 8x\,dx$. Use (3) with $n = -2$.

13. B. Rewrite and modify to $\frac{1}{8}\int (1 + 4x^2)^{-1/2} \cdot 8x\,dx$. Use (3) with $n = -\frac{1}{2}$.
Note carefully the differences in the integrands in questions 10 through 13.

14. C. Use (17). Note that $u = y$, $a = 2$.

15. B. Rewrite and modify to $-\frac{1}{2}\int (4 - y^2)^{-1/2} \cdot -2y\,dy$. Use (3).
Compare the integrands in questions 14 and 15, noting the difference.

16. A. Divide to obtain $\int \left(1 + \frac{1}{2} \cdot \frac{1}{x}\right)dx$. Use (2), (3), and (4). Remember that $\int k\,dx = kx + C$ whenever $k \neq 0$.

17. E. $\int \frac{(x-2)^3}{x^2}\,dx = \int \left(x - 6 + \frac{12}{x} - \frac{8}{x^2}\right)dx = \frac{x^2}{2} - 6x + 12 \ln|x| + \frac{8}{x} + C$. We used the binomial theorem with $n = 3$ on page 587 to expand $(x - 2)^3$.

18. D. The integral is equivalent to $\int \left(t - 2 + \frac{1}{t}\right)dt$. Integrate term by term.

19. A. Integrate term by term.

20. D. Long division yields

$$\int \left((x-2) + \frac{2x+1}{x^2+2x+1}\right)dx = \frac{1}{2}(x-2)^2 + \int \frac{2x+2-1}{x^2+2x+1}\,dx.$$

The integral equals $\int \frac{2x+2}{x^2+2x+1}\,dx - \int \frac{1}{(x+1)^2}\,dx$. Use formula (4) for the first integral, (3) for the second with $u = x + 1$ and $n = -2$. Note that $\ln|x+1|^2 = 2\ln|x+1|$.

21. E. Use formula (4) with $u = 1 - \sqrt{y} = 1 - y^{1/2}$. Then $du = -\frac{1}{2\sqrt{y}}\,dy$. Note that the integral can be written as $-2\int \frac{1}{(1-\sqrt{y})}\left(-\frac{1}{2\sqrt{y}}\right)dy$.

22. E. Use formula (3) after multiplying outside the integral by $-\frac{1}{18}$, inside by -18.

23. B. The integral is equal to $\frac{1}{2}\int \sin 2\theta\,d\theta$. Use formula (6) with $u = 2\theta$; $du = 2\,d\theta$.

24. C. Use formula (6) with $u = \sqrt{x}$; $du = \frac{1}{2\sqrt{x}}\,dx$.

25. A. Use formula (5) with $u = 4t^2$; $du = 8t\,dt$.

26. A. Using the half-angle formula (23) on page 589 with $\alpha = 2x$ yields $\int \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)dx$.

27. E. Use formula (6).

28. B. Integrate by parts (page 159). Let $u = x$ and $v' = \cos x$. Then $u' = 1$ and $v = \sin x$. The given integral equals $x \sin x - \int \sin x\,dx$.

29. D. Replace $\frac{1}{\cos^2 3u}$ by $\sec^2 3u$; then use formula (9).

30. C. The integral is of the form $\int \frac{du}{\sqrt{u}}$, where $u = 1 + \sin x$ and $du = \cos x\,dx$. Use formula (3) with $n = -\frac{1}{2}$.

31. B. The integral is equivalent to $\int \csc(\theta - 1)\cot(\theta - 1)\,d\theta$. Use formula (12).

32. E. Use formula (13) with $u = \frac{t}{2}$; $du = \frac{1}{2}\,dt$.

33. A. If we replace $\sin 2x$ by $2 \sin x \cos x$, the integral is equivalent to

$$-\int \frac{-2\sin x \cos x}{\sqrt{1+\cos^2 x}}\,dx = -\int \frac{du}{\sqrt{u}},$$

where $u = 1 + \cos^2 x$ and $du = -2 \sin x \cos x\,dx$. Use formula (3).

34. D. Rewriting in terms of sines and cosines yields

$$\int \frac{\sin x}{\cos^{5/2} x}\,dx = -\int \cos^{-5/2} x(-\sin x)dx = -\left(-\frac{2}{3}\right)\cos^{-3/2} x + C$$

35. E. Use formula (7).

36. C. Replace $\frac{1}{\sin^2 2x}$ by $\csc^2 2x$ and use formula (10).

37. E. If we let $u = \tan^{-1} y$, then we integrate $\int u\,du$. The correct answer is $\frac{1}{2}(\tan^{-1} y)^2 + C$.

38. A. Rewrite:

$$\int \sin^3 \theta(1 - \sin^2 \theta)\cos\theta\,d\theta = \int (\sin^3 \theta - \sin^5 \theta)\cos\theta\,d\theta.$$

39. C. The answer is equivalent to $\frac{1}{2}\ln|1-\cos 2t| + C$.

40. B. To use formula (8), multiply outside the integral by $\frac{1}{2}$, inside by 2.

41. E. Use formula (4) with $u = e^x - 1$; $du = e^x\,dx$.

42. D. Use partial fractions; find A and B such that

$$\frac{x-1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}.$$

Then $x - 1 = A(x-2) + Bx$.
Set $x = 0$: $-1 = -2A$ and $A = \frac{1}{2}$.
Set $x = 2$: $1 = 2B$ and $B = \frac{1}{2}$.
So the given integral equals

$$\int\left(\frac{1}{2x} + \frac{1}{2(x-2)}\right)dx = \frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + C$$
$$= \frac{1}{2}\ln|x(x-2)| + C.$$

43. A. Use formula (15) with $u = x^2$; $du = 2x\,dx$.

44. B. Use formula (15) with $u = \sin\theta$; $du = \cos\theta\,d\theta$.

45. C. Use formula (6) with $u = e^{2\theta}$; $du = 2e^{2\theta}\,d\theta$.

46. B. Use formula (15) with $u = \sqrt{x} = x^{1/2}$; $du = \frac{1}{2\sqrt{x}}\,dx$.

47. C. Use the Parts Formula. Let $u = x$, $v' = e^{-x}$; $u' = 1$, $v = -e^{-x}$. We get

$$-xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C.$$

48. C. See Example 49, page 159.

49. D. The integral is of the form $\int \frac{du}{u}$; use (4).

50. A. The integral has the form $\int \frac{du}{1+u^2}$. Use formula (18), with $u = e^x$, $du = e^x\,dx$, and $a = 1$.

51. C. Let $u = \ln v$; then $du = \frac{dv}{v}$. Use formula (3).

52. E. Hint: $\ln\sqrt{x} = \frac{1}{2}\ln x$.

53. B. Use parts, letting $u = \ln x$ and $v' = x^3$. Then $u' = \frac{1}{x}$ and $v = \frac{x^4}{4}$. The integral equals $\frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx$.

54. B. Use parts, letting $u = \ln\eta$ and $v' = 1$. Then $u' = \frac{1}{\eta}$ and $v = \eta$. The integral equals $\eta\ln\eta - \int d\eta$.

55. B. Rewrite $\ln x^3$ as $3\ln x$, and use the method of Answer 54.

56. D. Use parts, letting $u = \ln y$ and $v' = y^{-2}$. Then $u' = \frac{1}{y}$ and $v = -\frac{1}{y}$. The Parts Formula yields $\frac{-\ln y}{y} + \int \frac{1}{y^2}\,dy$.

57. E. The integral has the form $\int \frac{du}{u}$, where $u = \ln v$.

58. A. Hint: The integrand is equivalent to $1 - \frac{2}{y+1}$.

59. B. Hint: Let $u = \sqrt{t+1}$. Then

$$u^2 = t + 1, \qquad 2u\,du = dt, \qquad \text{and} \qquad t = u^2 - 1.$$

60. D. Hint: Multiply.

61. C. See Example 50, page 159. Replace x by θ.

62. E. The integral equals $-\int(1-\ln t)^2\left(-\frac{1}{t}\,dt\right)$; it is equivalent to $-\int u^2\,du$, where $u = 1 - \ln t$.

63. A. Replace u by x in the given integral to avoid confusion in applying the Parts Formula. To integrate $\int x\sec^2 x\,dx$, let the variable u in the Parts Formula be x, and let v' be $\sec^2 x\,dx$. Then $u' = 1$ and $v = \tan x$. So,

$$\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$$
$$= x\tan x + \ln|\cos x| + C.$$

64. D. The integral is equivalent to $\int \frac{2x}{4+x^2}\,dx + \int \frac{1}{4+x^2}\,dx$. Use formula (4) on the first integral and (18) on the second.

65. C. Rewrite:

$$\frac{1}{2}\int\frac{2x+4}{x^2+2x+10}\,dx = \frac{1}{2}\int\frac{2x+2+2}{x^2+2x+10}\,dx$$
$$= \frac{1}{2}\int\frac{2x+2}{x^2+2x+10}\,dx + \int\frac{dx}{(x+1)^2+3^2}\,dx.$$

Use formulas (4) and (18).

66. E. Hint: Letting $u = 4x - 4x^2$, we see that we are integrating $-\frac{1}{4}\int u^{-1/2}\,du$.

67. B. Hint: Divide, getting $\int\left[e^x - \frac{e^x}{1+e^x}\right]dx$.

68. D. Letting $u = \sin\theta$ yields the integral $\int\frac{du}{1+u^2}$. Use formula (18).

69. E. Use integration by parts, letting $u = \arctan x$ and $v' = dx$. Then

$$du = \frac{dx}{1+x^2} \quad \text{and} \quad v = x.$$

The Parts Formula yields $x\arctan x - \int\frac{x\,dx}{1+x^2}$, or

$$x\arctan x - \frac{1}{2}\ln(1+x^2) + C.$$

70. B. Hint: Note that

$$\frac{1}{1-e^x} = \frac{1-e^x+e^x}{1-e^x} = 1+\frac{e^x}{1-e^x}.$$

Or multiply the integrand by $\frac{e^{-x}}{e^{-x}}$, recognizing that the correct answer is equivalent to $-\ln\left|e^{-x}-1\right|$.

71. D. Hint: Expand the numerator and divide. Then integrate term by term.

72. C. Hint: Observe that $e^{2\ln u} = u^2$.

73. A. If we let $u = 1 + \ln y^2 = 1 + 2\ln|y|$, we want to integrate $\frac{1}{2}\int \frac{du}{u}$.

74. B. Hint: Expand and note that

$$\int(\tan^2\theta - 2\tan\theta + 1)\,d\theta = \int\sec^2\theta\,d\theta - 2\int\tan\theta\,d\theta.$$

Use formulas (9) and (7).

75. E. Multiply by $\frac{1-\sin\theta}{1-\sin\theta}$. The correct answer is $\tan\theta - \sec\theta + C$.

76. D. Note the initial conditions: when $t = 0$, $v = 0$ and $s = 0$. Integrate twice: $v = 6t^2$ and $s = 2t^3$. Let $t = 3$.

77. D. Since $y' = x^2 - 2$, $y = \frac{1}{3}x^3 - 2x + C$. Replacing x by 1 and y by -3 yields $C = -\frac{4}{3}$.

78. D. When $t = 0$, $v = 3$ and $s = 2$. So

$$v = 2t + 3t^2 + 3 \qquad \text{and} \qquad s = t^2 + t^3 + 3t + 2.$$

Let $t = 1$.

79. B. Let $a = \frac{dv}{dt} = -k$; then

$$v = -kt + C. \qquad (*)$$

Since $v = 75$ when $t = 0$, we get $C = 75$. Then (*) becomes

$$v = -kt + 75$$

so

$$0 = -5k + 75 \qquad \text{and} \qquad k = 15.$$

80. A. Divide to obtain $\int\left(1+\frac{1}{x^2-1}\right)dx$. Use partial fractions to get

$$\frac{1}{x^2-1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}.$$

Set 6: Multiple-Choice Questions on Definite Integrals

1. $\int_{-1}^{1} (x^2 - x - 1)\,dx =$

(A) $\frac{2}{3}$ **(B)** 0 **(C)** $-\frac{4}{3}$ **(D)** –2 **(E)** –1

2. $\int_{1}^{2} \frac{3x-1}{3x}\,dx =$

(A) $\frac{3}{4}$ **(B)** $1 - \frac{1}{3}\ln 2$ **(C)** $1 - \ln 2$ **(D)** $-\frac{1}{3}\ln 2$ **(E)** 1

3. $\int_{0}^{3} \frac{dt}{\sqrt{4-t}} =$

(A) 1 **(B)** –2 **(C)** 4 **(D)** –1 **(E)** 2

4. $\int_{-1}^{0} \sqrt{3u+4}\,du =$

(A) 2 **(B)** $\frac{14}{9}$ **(C)** $\frac{14}{3}$ **(D)** 6 **(E)** $\frac{7}{2}$

5. $\int_{2}^{3} \frac{dy}{2y-3} =$

(A) $\ln 3$ **(B)** $\frac{1}{2}\ln\frac{3}{2}$ **(C)** $\frac{16}{9}$ **(D)** $\ln\sqrt{3}$ **(E)** $\sqrt{3} - 1$

6. $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx =$

(A) 1 **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** –1 **(E)** 2

7. $\int_{0}^{1} (2t-1)^3\,dt =$

(A) $\frac{1}{4}$ **(B)** 6 **(C)** $\frac{1}{2}$ **(D)** 0 **(E)** 4

8. $\int_{0}^{1} \frac{dx}{\sqrt{4-x^2}}$ equals, to three decimal places

(A) 0.262 **(B)** 0.268 **(C)** 0.524 **(D)** 0.536 **(E)** 1.047

9. $\int_{4}^{9} \frac{2+x}{2\sqrt{x}}\,dx =$

(A) $\frac{25}{3}$ **(B)** $\frac{41}{3}$ **(C)** $\frac{100}{3}$ **(D)** $\frac{5}{3}$ **(E)** $\frac{1}{3}$

10. $\int_{-3}^{3} \frac{dx}{9+x^2} =$

(A) $\frac{\pi}{2}$ **(B)** 0 **(C)** $\frac{\pi}{6}$ **(D)** $-\frac{\pi}{2}$ **(E)** $\frac{\pi}{3}$

11. $\int_{0}^{1} e^{-x}\,dx =$

(A) $\frac{1}{e} - 1$ **(B)** $1 - e$ **(C)** $-\frac{1}{e}$ **(D)** $1 - \frac{1}{e}$ **(E)** $\frac{1}{e}$

12. $\int_{0}^{1} xe^{x^2}\,dx =$

(A) $e - 1$ **(B)** $\frac{1}{2}(e-1)$ **(C)** $2(e-1)$ **(D)** $\frac{e}{2}$ **(E)** $\frac{e}{2} - 1$

13. $\int_{0}^{\pi/4} \sin 2\theta\,d\theta =$

(A) 2 **(B)** $\frac{1}{2}$ **(C)** –1 **(D)** $-\frac{1}{2}$ **(E)** –2

14. $\int_{1}^{2} \frac{dz}{3-z} =$

(A) $-\ln 2$ **(B)** $\frac{3}{4}$ **(C)** $2(\sqrt{2} - 1)$ **(D)** $\frac{1}{2}\ln 2$ **(E)** $\ln 2$

***15.** If we let $x = 2\sin\theta$ then $\int_{1}^{2} \frac{\sqrt{4-x^2}}{x}\,dx$ is equivalent to

(A) $2\int_{0}^{2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$ **(B)** $\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin\theta}\,d\theta$ **(C)** $2\int_{\pi/6}^{\pi/6} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

(D) $\int_{1}^{2} \frac{\cos\theta}{\sin\theta}\,d\theta$ **(E)** none of these

*An asterisk denotes a topic covered only in Calculus BC.

16. The integral $\int_{-4}^{4} \sqrt{16-x^2}\,dx$ gives the area of

(A) a circle of radius 4
(B) a semicircle of radius 4
(C) a quadrant of a circle of radius 4
(D) an ellipse whose semimajor axis is 4
(E) none of these

17. $\int_0^{\pi} \cos^2\theta \sin\theta\, d\theta =$

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) 0

18. $\int_1^e \frac{\ln x}{x}\,dx =$

(A) $\frac{1}{2}$ (B) $\frac{1}{2}(e^2-1)$ (C) 0 (D) 1 (E) $e-1$

*19. $\int_0^1 xe^x\,dx =$

(A) -1 (B) $e+1$ (C) 1 (D) $e-1$ (E) $\frac{1}{2}(e-1)$

20. $\int_0^{\pi/6} \frac{\cos\theta}{1+2\sin\theta}\,d\theta =$

(A) $\ln 2$ (B) $\frac{3}{8}$ (C) $-\frac{1}{2}\ln 2$ (D) $\frac{3}{2}$ (E) $\ln\sqrt{2}$

21. $\int_0^{\pi/4} \sqrt{1-\cos 2\alpha}\,d\alpha =$

(A) 0.25 (B) 0.414 (C) 1.000 (D) 1.414 (E) 2.000

22. $\int_{\sqrt{2}}^{2} \frac{u}{u^2-1}\,du =$

(A) $\ln\sqrt{3}$ (B) $\frac{8}{9}$ (C) $\ln\frac{3}{2}$ (D) $\ln 3$ (E) $1-\sqrt{3}$

*An asterisk denotes a topic covered only in Calculus BC.

23. $\int_{\sqrt{2}}^{2} \frac{u\,du}{(u^2-1)^2} =$

(A) $-\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) -1 (E) $\frac{1}{3}$

24. $\int_0^{\pi/4} \cos^2\theta\,d\theta =$

(A) $\frac{1}{2}$ (B) $\frac{\pi}{8}$ (C) $\frac{\pi}{8}+\frac{1}{4}$ (D) $\frac{\pi}{8}+\frac{1}{2}$ (E) $\frac{\pi}{8}-\frac{1}{4}$

25. $\int_{\pi/12}^{\pi/4} \frac{\cos 2x\,dx}{\sin^2 2x} =$

(A) $-\frac{1}{4}$ (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) -1

26. $\int_0^1 \frac{e^{-x}+1}{e^{-x}}\,dx =$

(A) e (B) $2+e$ (C) $\frac{1}{e}$ (D) $1+e$ (E) $e-1$

27. $\int_0^1 \frac{e^x}{e^x+1}\,dx =$

(A) $\ln 2$ (B) e (C) $1+e$ (D) $-\ln 2$ (E) $\ln\frac{e+1}{2}$

28. If $f(x)$ is continuous on the interval $a \le x \le b$ and $a < c < b$, then $\int_c^b f(x)\,dx$ is equal to

(A) $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ (B) $\int_a^c f(x)\,dx - \int_a^b f(x)\,dx$

(C) $\int_c^a f(x)\,dx + \int_b^a f(x)\,dx$ (D) $\int_a^b f(x)\,dx - \int_a^c f(x)\,dx$

(E) $\int_a^c f(x)\,dx - \int_b^c f(x)\,dx$

29. If $f(x)$ is continuous on $a \le x \le b$, then

(A) $\int_a^b f(x)\,dx = f(b) - f(a)$ (B) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(C) $\int_a^b f(x)\,dx \geqq 0$ (D) $\frac{d}{dx}\int_a^x f(t)\,dt = f'(x)$

(E) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x) - f(a)$

30. If $f(x)$ is continuous on the interval $a \leqq x \leqq b$, if this interval is partitioned into n equal subintervals of length Δx, and if x_k is a number in the kth subinterval, then $\lim_{n\to\infty} \sum_{1}^{n} f(x_k)\ \Delta x$ is equal to

(A) $f(b) - f(a)$
(B) $F(x) + C$, where $\frac{dF(x)}{dx} = f(x)$ and C is an arbitrary constant
(C) $\int_a^b f(x)\,dx$
(D) $F(b-a)$, where $\frac{dF(x)}{dx} = f(x)$
(E) none of these

31. If $F'(x) = G'(x)$ for all x, then

(A) $\int_a^b F'(x)\,dx = \int_a^b G'(x)\,dx$ **(B)** $\int F(x)\,dx = \int G(x)\,dx$
(C) $\int_a^b F(x)\,dx = \int_a^b G(x)\,dx$ **(D)** $\int F(x)\,dx = \int G(x)\,dx + C$
(E) none of the preceding is necessarily true

32. If $f(x)$ is continuous on the closed interval $[a,b]$, then there exists at least one number c, $a < c < b$, such that $\int_a^b f(x)\,dx$ is equal to

(A) $\frac{f(c)}{b-a}$ **(B)** $f'(c)(b-a)$ **(C)** $f(c)(b-a)$
(D) $\frac{f'(c)}{b-a}$ **(E)** $f(c)[f(b)-f(a)]$

33. If $f(x)$ is continuous on the closed interval $[a,b]$ and k is a constant, then $\int_a^b kf(x)\,dx$ is equal to

(A) $k(b-a)$ **(B)** $k[f(b)-f(a)]$ **(C)** $kF(b-a)$, where $\frac{dF(x)}{dx} = f(x)$
(D) $k\int_a^b f(x)\,dx$ **(E)** $\left.\frac{[kf(x)]^2}{2}\right]_a^b$

34. $\frac{d}{dt}\int_0^t \sqrt{x^3+1}\,dx =$

(A) $\sqrt{t^3+1}$ **(B)** $\frac{\sqrt{t^3+1}}{3t^2}$ **(C)** $\frac{2}{3}(t^3+1)(\sqrt{t^3+1}-1)$
(D) $3x^2\sqrt{x^3+1}$ **(E)** none of these

35. If $F(u) = \int_1^u (2-x^2)^3\,dx$, then $F'(u)$ is equal to

(A) $-6u(2-u^2)^2$ **(B)** $\frac{(2-u^2)^4}{4} - \frac{1}{4}$ **(C)** $(2-u^2)^3 - 1$
(D) $(2-u^2)^3$ **(E)** $-2u(2-u^2)^3$

36. $\frac{d}{dx}\int_{\pi/2}^{x^2} \sqrt{\sin t}\,dt =$

(A) $\sqrt{\sin t^2}$ **(B)** $2x\sqrt{\sin x^2} - 1$ **(C)** $\frac{2}{3}(\sin^{3/2} x^2 - 1)$
(D) $\sqrt{\sin x^2} - 1$ **(E)** $2x\sqrt{\sin x^2}$

37. If we let $x = \tan\theta$, then $\int_1^{\sqrt{3}} \sqrt{1+x^2}\,dx$ is equivalent to

(A) $\int_{\pi/4}^{\pi/3} \sec\theta\,d\theta$ **(B)** $\int_1^{\sqrt{3}} \sec^3\theta\,d\theta$ **(C)** $\int_{\pi/4}^{\pi/3} \sec^3\theta\,d\theta$
(D) $\int_{\pi/4}^{\pi/3} \sec^2\theta\tan\theta\,d\theta$ **(E)** $\int_1^{\sqrt{3}} \sec\theta\,d\theta$

38. If the substitution $u = \sqrt{x+1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2 \frac{du}{u^2-1}$ **(B)** $\int_1^2 \frac{2\,du}{u^2-1}$ **(C)** $2\int_0^3 \frac{du}{(u-1)(u+1)}$
(D) $2\int_1^2 \frac{du}{u(u^2-1)}$ **(E)** $2\int_0^3 \frac{du}{u(u-1)}$

39. If $x = 4\cos\theta$ and $y = 3\sin\theta$, then $\int_2^4 xy\,dx$ is equivalent to

(A) $48\int_{\pi/3}^{0} \sin\theta\cos^2\theta\,d\theta$ **(B)** $48\int_2^4 \sin^2\theta\cos\theta\,d\theta$
(C) $36\int_2^4 \sin\theta\cos^2\theta\,d\theta$ **(D)** $-48\int_0^{\pi/3} \sin\theta\cos^2\theta\,d\theta$
(E) $48\int_0^{\pi/3} \sin^2\theta\cos\theta\,d\theta$

***40.** A curve is defined by the parametric equations $y = 2a\cos^2\theta$ and $x = 2a\tan\theta$, where $0 \leqq \theta \leqq \pi$. Then the definite integral $\pi\int_0^{2a} y^2\,dx$ is equivalent to

(A) $4\pi a^2\int_0^{\pi/4}\cos^4\theta\,d\theta$ **(B)** $8\pi a^3\int_{\pi/2}^{\pi}\cos^2\theta\,d\theta$ **(C)** $8\pi a^3\int_0^{\pi/4}\cos^2\theta\,d\theta$

(D) $8\pi a^3\int_0^{2a}\cos^2\theta\,d\theta$ **(E)** $8\pi a^3\int_0^{\pi/4}\sin\theta\cos^2\theta\,d\theta$

***41.** A curve is given parametrically by $x = 1 - \cos t$ and $y = t - \sin t$, where $0 \leqq t \leqq \pi$. Then $\int_0^{3/2} y\,dx$ is equivalent to

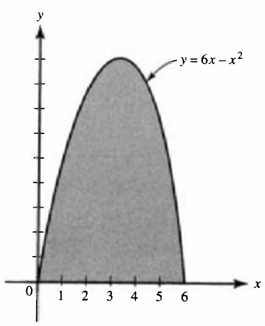
(A) $\int_0^{3/2}\sin t(t - \sin t)\,dt$ **(B)** $\int_{2\pi/3}^{\pi}\sin t(t - \sin t)\,dt$

(C) $\int_0^{2\pi/3}(t - \sin t)\,dt$ **(D)** $\int_0^{2\pi/3}\sin t(t - \sin t)\,dt$

(E) $\int_0^{3/2}(t - \sin t)\,dt$

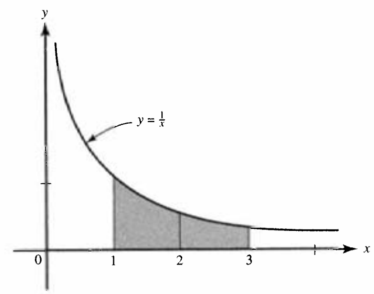
42. If we approximate the area of the shaded region by $M(20)$ (that is, the midpoint sum with 20 subintervals), then the difference $M(20) - \int_0^6 f(x)\,dx$ is equal to

(A) 0.004 **(B)** 0.008 **(C)** 0.010
(D) 0.045 **(E)** none of these



*An asterisk denotes a topic covered only in Calculus BC.

43. The area of the following shaded region is equal exactly to ln 3. If we approximate ln 3 using $L(2)$ and $R(2)$, which of the inequalities follows?



(A) $\frac{1}{2} < \int_1^2 \frac{1}{x}dx < 1$ **(B)** $\frac{1}{3} < \int_1^3 \frac{1}{x}dx < 2$ **(C)** $\frac{1}{2} < \int_0^2 \frac{1}{x}dx < 2$

(D) $\frac{1}{3} < \int_2^3 \frac{1}{x}dx < \frac{1}{2}$ **(E)** $\frac{5}{6} < \int_1^3 \frac{1}{x}dx < \frac{3}{2}$

44. $\int_0^2 \sqrt{3x+1}\,dx$ is equal approximately to

(A) 3.646 **(B)** 3.893 **(C)** 4.116 **(D)** 4.646 **(E)** 11.680

45. $\int_1^2 \frac{dx}{\sqrt{4-x^2}}$ is equal, to three decimal places, approximately to

(A) 0.322 **(B)** 0.524 **(C)** 1.047
(D) 1.570 **(E)** none of these

46. If the Trapezoidal Rule is used with $n = 5$, then $\int_0^1 \frac{dx}{1+x^2}$ is equal, to three decimal places, to

(A) 0.784 **(B)** 1.567 **(C)** 1.959 **(D)** 3.142 **(E)** 7.837

47. $\int_{-1}^3 |x|\,dx =$

(A) $\frac{7}{2}$ **(B)** 4 **(C)** $\frac{9}{2}$ **(D)** 5 **(E)** $\frac{11}{2}$

48. $\int_{-3}^{2} |x+1|\, dx =$

(A) $\frac{5}{2}$ **(B)** $\frac{7}{2}$ **(C)** 5 **(D)** $\frac{11}{2}$ **(E)** $\frac{13}{2}$

49. If $M(4)$ is used to approximate $\int_0^1 \sqrt{1+x^3}\, dx$, then the definite integral is equal, to two decimal places, to

(A) 1.00 **(B)** 1.11 **(C)** 1.20 **(D)** 2.22 **(E)** 3.33

50. $\int_2^5 \frac{1}{3x}\, dx$ is best approximated, to three decimal places, by

(A) 0.268 **(B)** 0.286 **(C)** 0.305 **(D)** 0.916 **(E)** 2.749

51. The average value of $\cos x$ over the interval $\frac{\pi}{3} \leqq x \leqq \frac{\pi}{2}$ is

(A) $\frac{3}{\pi}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3(2-\sqrt{3})}{\pi}$ **(D)** $\frac{3}{2\pi}$ **(E)** $\frac{2}{3\pi}$

52. The average value of $\csc^2 x$ over the interval from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$ is

(A) $\frac{3\sqrt{3}}{\pi}$ **(B)** $\frac{\sqrt{3}}{\pi}$ **(C)** $\frac{12}{\pi}(\sqrt{3}-1)$

(D) $3\sqrt{3}$ **(E)** $3(\sqrt{3}-1)$

Answers for Set 6: Definite Integrals

1.	C	**12.**	B	**23.**	E	**34.**	A	**45.**	C
2.	B	**13.**	B	**24.**	C	**35.**	D	**46.**	A
3.	E	**14.**	E	**25.**	C	**36.**	E	**47.**	D
4.	B	**15.**	C	**26.**	A	**37.**	C	**48.**	E
5.	D	**16.**	B	**27.**	E	**38.**	B	**49.**	B
6.	A	**17.**	D	**28.**	D	**39.**	E	**50.**	C
7.	D	**18.**	A	**29.**	B	**40.**	C	**51.**	C
8.	C	**19.**	C	**30.**	C	**41.**	D	**52.**	C
9.	A	**20.**	E	**31.**	A	**42.**	D		
10.	C	**21.**	B	**32.**	C	**43.**	E		
11.	D	**22.**	A	**33.**	D	**44.**	B		

1. C. The integral is equal to

$$\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - x\right)\Bigg|_{-1}^{1} = -\frac{7}{6} - \frac{1}{6}.$$

2. B. Rewrite as $\int_1^2 \left(1 - \frac{1}{3}\cdot\frac{1}{x}\right) dx$. This equals

$$\left(x - \frac{1}{3}\ln x\right)\Bigg|_1^2 = 2 - \frac{1}{3}\ln 2 - 1.$$

3. E. Rewrite as

$$-\int_0^3 (4-t)^{-1/2}(-1\, dt) = -2\sqrt{4-t}\,\Bigg|_0^3 = -2(1-2).$$

4. B. This one equals

$$\frac{1}{3}\int_{-1}^{0} (3u+4)^{1/2}\cdot 3\, du = \frac{1}{3}\cdot\frac{2}{3}(3u+4)^{3/2}\Bigg|_{-1}^{0}$$
$$= \frac{2}{9}(4^{3/2} - 1^{3/2}).$$

5. D. We have:

$$\frac{1}{2}\int_2^3 \frac{2\,dy}{2y-3} = \frac{1}{2}\ln(2y-3)\Bigg|_2^3 = \frac{1}{2}(\ln 3 - \ln 1).$$

6. A. Rewrite:

$$-\frac{1}{2}\int_0^{\sqrt{3}} (4-x^2)^{-1/2}(-2x\,dx) = -\frac{1}{2}\cdot 2\sqrt{4-x^2}\,\Big|_0^{\sqrt{3}} = -(1-2).$$

7. D. We expand, then integrate term by term:

$$(8t^3 - 12t^2 + 6t - 1)\Big|_0^1 = (2t^4 - 4t^3 + 3t^2 - t)\Big|_0^1 = (2-4+3-1) = 0.$$

8. C. Using a graphing calculator. Compare with Question 6.

9. A. We divide:

$$\int_4^9 \left(x^{-1/2} + \frac{1}{2}x^{1/2}\right)dx = \left(2x^{1/2} + \frac{1}{2}\cdot\frac{2}{3}x^{3/2}\right)\Big|_4^9$$
$$= \left(2\cdot 3 + \frac{1}{3}\cdot 27\right) - \left(2\cdot 2 + \frac{1}{3}\cdot 8\right).$$

10. C. The integral equals

$$\frac{1}{3}\tan^{-1}\frac{x}{3}\Big|_{-3}^{3} = \frac{1}{3}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right).$$

11. D. We get $-e^{-x}\Big|_0^1 = -(e^{-1} - 1).$

12. B. The integral equals $\frac{1}{2}e^x\Big|_0^1 = \frac{1}{2}(e^1 - 1).$

13. B. We evaluate $-\frac{1}{2}\cos 2\theta\Big|_0^{\pi/4}$, which equals $-\frac{1}{2}(0-1).$

14. E. We evaluate $-\ln(3-z)\Big|_1^2$ and get $-(\ln 1 - \ln 2).$

15. C. If $x = 2\sin\theta$, $\sqrt{4-x^2} = 2\cos\theta$, $dx = 2\cos\theta\,d\theta$. When $x = 1$, $\theta = \frac{\pi}{6}$; when $x = 2$, $\theta = \frac{\pi}{2}$. So, the integral is equivalent to $\int_{\pi/6}^{\pi/2} \frac{(2\cos\theta)(2\cos\theta)\,d\theta}{2\sin\theta}$.

16. B. The given integral equals the area of a semicircle of radius 4.

17. D. Evaluate $-\int_0^{\pi}\cos^2\theta(-\sin\theta)\,d\theta$. This equals $-\frac{1}{3}\cos^3\theta\Big|_0^{\pi} = -\frac{1}{3}(-1-1).$

18. A. The integral equals $\frac{1}{2}\ln^2 x\Big|_1^e = \frac{1}{2}(1-0).$

19. C. We use the parts formula with $u = x$ and $dv = e^x\,dx$. Then $du = dx$ and $v = e^x$. We get

$$\left(xe^x - \int e^x dx\right)\Big|_0^1 = (xe^x - e^x)\Big|_0^1 = (e-e) - (0-1).$$

20. E. We evaluate $\frac{1}{2}\ln(1 + 2\sin\theta)\Big|_0^{\pi/6}$ and get $\frac{1}{2}(\ln(1+1) - \ln 1).$

21. B. Use a graphing calculator.

22. A. Evaluate the integral $\frac{1}{2}\int_{\sqrt{2}}^{2}\frac{2u}{u^2-1}\,du$. It equals

$$\frac{1}{2}\ln(u^2-1)\Big|_{\sqrt{2}}^{2} \quad \text{or} \quad \frac{1}{2}(\ln 3 - \ln 1).$$

23. E. We evaluate $\frac{1}{2}\int_{\sqrt{2}}^{2}(u^2-1)^{-2}\cdot 2u\,du$ and get

$$-\frac{1}{2(u^2-1)}\Big|_{\sqrt{2}}^{2} \quad \text{or} \quad -\frac{1}{2}\left(\frac{1}{3} - \frac{1}{1}\right).$$

Compare with question 22 above.

24. C. Use the formula: $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$. Then

$$\frac{1}{2}\int_0^{\pi/4}(1+\cos 2\theta)\,d\theta = \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/4} = \frac{1}{2}\left(\left(\frac{\pi}{4} + \frac{1}{2}\right) - 0\right).$$

25. C. Rewrite:

$$\frac{1}{2}\int_{\pi/12}^{\pi/4}\sin^{-2}2x\cos 2x\,(2\,dx) = -\frac{1}{2}\cdot\frac{1}{\sin 2x}\Big|_{\pi/12}^{\pi/4}$$
$$= -\frac{1}{2}\left(\frac{1}{1} - \frac{1}{1/2}\right).$$

26. A. The integral is equivalent to

$$\int_0^1 (1 + e^x)\,dx = (x + e^x)\Big|_0^1 = (1+e) - 1.$$

27. E. We evaluate $\ln(e^x + 1)\Big|_0^1$, getting $\ln(e+1) - \ln 2$.

28. D. Note that the integral from a to b is the sum of the two integrals from a to c and from c to b.

29. B. Find the errors in (A), (C), (D), and (E).

30. C. This is the definition of the definite integral given on page 181.

31. A. Find examples of functions F and G that show that (B), (C), and (D) are false.

32. C. This is the Mean Value Theorem for integrals (page 182). Draw some sketches that illustrate the theorem.

33. D. This is the theorem (2) on page 182. Prove by counterexamples that (A), (B), (C), and (D) are false.

34. A. This is a restatement of the Fundamental Theorem. In theorem (1) on page 182, interchange t and x.

35. D. Apply (1) on page 182, noting that

$$F'(u) = \frac{d}{du}\int_a^u f(x)\,dx = f(u).$$

36. E. If we let $y = \int_{\pi/2}^{x^2} \sqrt{\sin t}\,dt$ and $u = x^2$, then

$$y = \int_{\pi/2}^{u} \sqrt{\sin t}\,dt$$

By the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \sqrt{\sin u}\cdot 2x$, where we used theorem (1) on page 182 to find $\frac{dy}{du}$. Replace u by x^2.

37. C. Note that $dx = \sec^2\theta\,d\theta$ and that $\sqrt{1+\tan^2\theta} = \sec\theta$. Be sure to express the limits as values of θ: $1 = \tan\theta$ yields $\theta = \frac{\pi}{4}$; $\sqrt{3} = \tan\theta$ yields $\theta = \frac{\pi}{3}$.

38. B. If $u = \sqrt{x+1}$, then $u^2 = x + 1$, and $2u\,du = dx$. When we substitute for the limits we get $2\int_1^2 \frac{u\,du}{u(u^2-1)}$. Since $u \neq 0$ on its interval of integration, we may divide numerator and denominator by it.

39. E. Since $dx = -4\sin\theta\,d\theta$, we get the new integral $-48\int_{\pi/3}^{0} \sin^2\theta\cos\theta\,d\theta$. Use theorem (4) on page 182 to get the correct answer.

40. C. Since $dx = 2a\sec^2\theta\,d\theta$, we get $8\pi a^3\int_0^{\pi/4} \cos^4\theta\sec^2\theta\,d\theta$. Use the fact that $\cos^2\theta\sec^2\theta = 1$.

41. D. Use the facts that $dx = \sin t\,dt$, that $t = 0$ when $x = 0$, and that $t = \frac{2\pi}{3}$ when $x = \frac{3}{2}$.

42. D. $M(20) - \int_0^6 (6x - x^2)\,dx = 36.045 - 36 = 0.045.$

43. E. For $L(2)$ we use the circumscribed rectangles:

$$1\cdot 1 + \frac{1}{2}\cdot 1 = \frac{3}{2};$$

for $R(2)$ we use the inscribed rectangles:

$$\frac{1}{2}\cdot 1 + \frac{1}{3}\cdot 1 = \frac{5}{6}.$$

44. B. Using the `[fnInt]` program, we get choice (B) for the integral.

45. C. This is the answer given by our calculator.

46. A. $\int_0^1 \frac{dx}{1+x^2}$, calculated approximately by $T(5)$ on our graphing calculator yields choice A.

47. D We must rewrite the integral to evaluate it, using the fact that x changes sign at 0. We get

$$\int_{-1}^{0} (-x)\,dx + \int_0^3 x\,dx = -\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_0^3.$$

Draw a sketch of $y = |x|$ and verify that the area over $-1 \leqq x \leqq 3$ equals 5.

48. E. Since $x + 1$ changes sign at $x = -1$, $|x + 1| = -(x + 1)$ if $x < -1$ but equals $x + 1$ if $x \geqq -1$. The given integral is therefore equivalent to

$$\int_{-3}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx = -\frac{(x+1)^2}{2}\Big|_{-3}^{-1} + \frac{(x+1)^2}{2}\Big|_{-1}^{2}$$

$$= -\frac{1}{2}(0-4) + \frac{1}{2}(9-0).$$

Draw a sketch of $y = |x+1|$, and verify that the area over $-3 \leqq x \leqq 2$ is $\frac{13}{2}$.

49. B. Our calculator gives 1.11 when we round to two decimal places.

50. C. $\int_2^5 \frac{1}{3x}\,dx = 0.305$, using our `[fnInt]` program.

51. C. Equation (1) on page 201 yields

$$(y_{av})_x = \frac{1}{\pi/2 - \pi/3}\int_{\pi/3}^{\pi/2} \cos x\,dx.$$

52. C. The average value is equal to $\frac{1}{\pi/4 - \pi/6}\int_{\pi/6}^{\pi/4} \csc^2 x\,dx.$