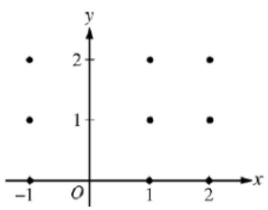
2008 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   (Note: Use the axes provided in the exam booklet.)



   (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

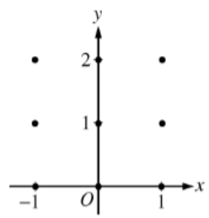
   (c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

   (Note: Use the axes provided in the exam booklet.)



   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

   (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

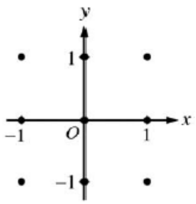
   (d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

2006 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

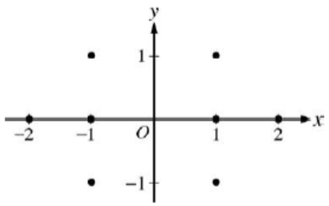
(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

2006 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)

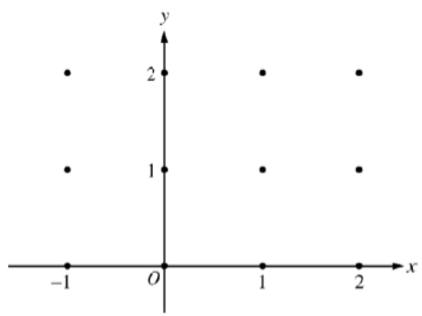


(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



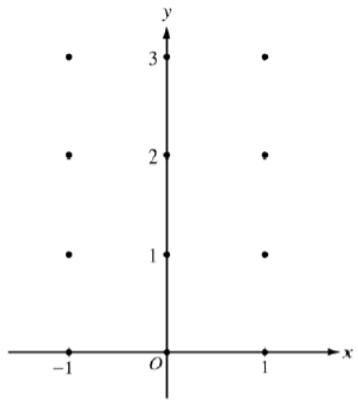
   (b) Write an equation for the line tangent to the graph of f at $x = -1$.

   (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

2004 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the pink test booklet.)**



   (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

2003 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

   (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

   (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

1998 AP Calculus AB Free-Response Questions

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

   (d) Use your solution from part (c) to find $f(1.2)$.

1997 AB6/BC6

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\dfrac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim\limits_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

1980 BC5

(a) Find the general solution of the differential equation $xy' + y = 0$.

(b) Find the general solution of the differential equation $xy' + y = 2x^2 y$.

(c) Find the particular solution of the differential equation in part (b) that satisfies the condition that $y = e^2$ when $x = 1$.

1993 AB6

Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

(a) If $P(0) = 500$, find $P(t)$ in terms of t and k.

(b) If $P(2) = 700$, find k.

(c) Find $\lim_{t \to \infty} P(t)$.

AP® CALCULUS AB
2008 SCORING GUIDELINES

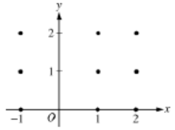
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

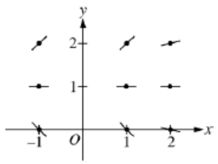
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.



(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) $\frac{1}{y-1}\,dy = \frac{1}{x^2}\,dx$

$\ln|y-1| = -\frac{1}{x} + C$

$|y-1| = e^{-\frac{1}{x}+C}$

$|y-1| = e^C e^{-\frac{1}{x}}$

$y - 1 = ke^{-\frac{1}{x}}$, where $k = \pm e^C$

$-1 = ke^{-\frac{1}{2}}$

$k = -e^{\frac{1}{2}}$

$f(x) = 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)}$, $x > 0$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antidifferentiates} \\ 1 : \text{includes constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration
Note: $0/6$ if no separation of variables

(c) $\lim_{x \to \infty} 1 - e^{\left(\frac{1}{2} - \frac{1}{x}\right)} = 1 - \sqrt{e}$

1 : limit

AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 5

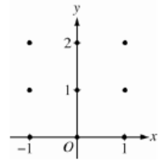
Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

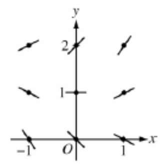
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.



(a)



2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\frac{1}{2}x + \frac{1}{2}$.

$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

(c) $\left.\frac{dy}{dx}\right|_{(0,1)} = 0 + 1 - 1 = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) Substituting $y = mx + b$ into the differential equation:

$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

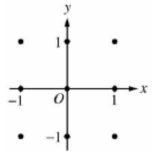
$2 : \begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$

AP® CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = (y-1)^2\cos(\pi x)$.

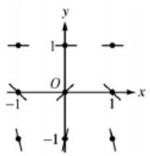
(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



$2: \begin{cases} 1: \text{zero slopes} \\ 1: \text{all other slopes} \end{cases}$

(b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

$1: c = 1$

(c)
$$\frac{1}{(y-1)^2}\,dy = \cos(\pi x)\,dx$$
$$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$$
$$1 = \frac{1}{\pi}\sin(\pi) + C = C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$$
$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$
$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \quad \text{for } -\infty < x < \infty$$

$6: \begin{cases} 1: \text{separates variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{answer} \end{cases}$

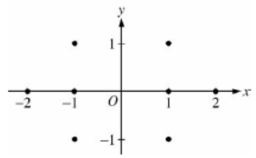
Note: max 3/6 [1-2-0-0-0] if no constant of integration
Note: 0/6 if no separation of variables

AP® CALCULUS AB
2006 SCORING GUIDELINES

Question 5

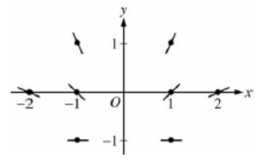
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
(Note: Use the axes provided in the pink exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b)
$$\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$$
$$\ln|1+y| = \ln|x| + K$$
$$|1+y| = e^{\ln|x| + K}$$
$$1 + y = C|x|$$
$$2 = C$$
$$1 + y = 2|x|$$
$y = 2|x| - 1$ and $x < 0$
or
$y = -2x - 1$ and $x < 0$

$7: \begin{cases} 6: \begin{cases} 1: \text{separates variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases} \\ \text{Note: max 3/6 [1-2-0-0-0] if no constant of integration} \\ \text{Note: 0/6 if no separation of variables} \\ 1: \text{domain} \end{cases}$

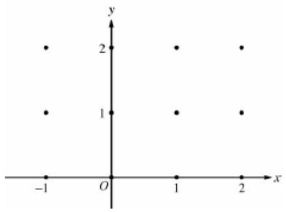
AP® CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

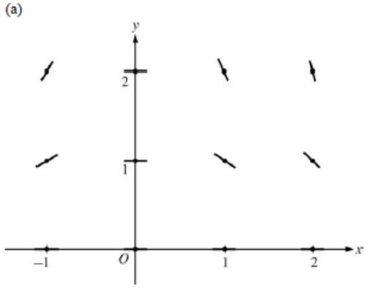
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) Write an equation for the line tangent to the graph of f at $x = -1$.



(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) Slope $= \frac{-(-1)4}{2} = 2$

$y - 2 = 2(x + 1)$

1 : equation

(c) $\frac{1}{y^2}dy = -\frac{x}{2}dx$

$-\frac{1}{y} = -\frac{x^2}{4} + C$

$-\frac{1}{2} = -\frac{1}{4} + C;\ C = -\frac{1}{4}$

$y = \frac{1}{\frac{x^2}{4} + \frac{1}{4}} = \frac{4}{x^2 + 1}$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

AP® CALCULUS AB
2004 SCORING GUIDELINES

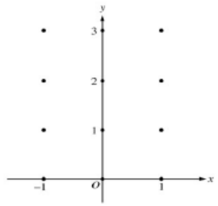
Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

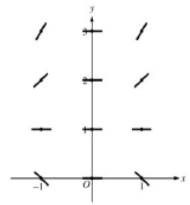
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



(a)



$2 : \begin{cases} 1 : \text{zero slope at each point } (x, y) \text{ where } x = 0 \text{ or } y = 1 \\ 1 : \begin{cases} \text{positive slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y > 1 \\ \text{negative slope at each point } (x, y) \text{ where } x \neq 0 \text{ and } y < 1 \end{cases} \end{cases}$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c) $\frac{1}{y-1}dy = x^2 dx$

$\ln|y - 1| = \frac{1}{3}x^3 + C$

$|y - 1| = e^C e^{\frac{1}{3}x^3}$

$y - 1 = Ke^{\frac{1}{3}x^3},\ K = \pm e^C$

$2 = Ke^0 = K$

$y = 1 + 2e^{\frac{1}{3}x^3}$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \\ \quad 0/1 \text{ if } y \text{ is not exponential} \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

AP® CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 6

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

(a) $f''(x) = \sqrt{f(x)} + x \cdot \frac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \frac{x^2}{2}$ $f''(3) = \sqrt{25} + \frac{9}{2} = \frac{19}{2}$	3 : { 2 : $f''(x)$ < −2 > product or chain rule error 1 : value at $x = 3$
(b) $\frac{1}{\sqrt{y}}\,dy = x\,dx$ $2\sqrt{y} = \frac{1}{2}x^2 + C$ $2\sqrt{25} = \frac{1}{2}(3)^2 + C\,;\; C = \frac{11}{2}$ $\sqrt{y} = \frac{1}{4}x^2 + \frac{11}{4}$ $y = \left(\frac{1}{4}x^2 + \frac{11}{4}\right)^2 = \frac{1}{16}\left(x^2 + 11\right)^2$	6 : { 1 : separates variables 1 : antiderivative of dy term 1 : antiderivative of dx term 1 : constant of integration 1 : uses initial condition $f(3) = 25$ 1 : solves for y Note: max 3/6 [1-1-1-0-0-0] if no constant of integration Note: 0/6 if no separation of variables

AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$ $\left.\frac{d^2y}{dx^2}\right\rvert_{(3,-2)} = \left.\frac{-y - y'(3 - x)}{y^2}\right\rvert_{(3,-2)} = \frac{1}{2}$, so f has a local minimum at this point. or Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which $y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the right of $x = 3$. Therefore f has a local minimum at $x = 3$.	3 { 1 : $x = 3$ 1 : local minimum 1 : justification
(b) $y\,dy = (3 - x)\,dx$ $\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$ $8 = 18 - 18 + C\,;\; C = 8$ $y^2 = 6x - x^2 + 16$ $y = -\sqrt{6x - x^2 + 16}$	6 { 1 : separates variables 1 : antiderivative of dy term 1 : antiderivative of dx term 1 : constant of integration 1 : uses initial condition $g(6) = -4$ 1 : solves for y Note: max 3/6 [1-1-1-0-0-0] if no constant of integration Note: 0/6 if no separation of variables

1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
   (c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.
   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ $\left.\frac{dy}{dx}\right\|_{\substack{x=1\\y=4}} = \frac{3+1}{2\cdot 4} = \frac{4}{8} = \frac{1}{2}$	**1**: answer
(b) $y - 4 = \frac{1}{2}(x-1)$ $f(1.2) - 4 \approx \frac{1}{2}(1.2-1)$ $f(1.2) \approx 0.1 + 4 = 4.1$	**2** { 1: equation of tangent line 1: uses equation to approximate $f(1.2)$
(c) $2y\,dy = (3x^2+1)\,dx$ $\int 2y\,dy = \int (3x^2+1)\,dx$ $y^2 = x^3 + x + C$ $4^2 = 1 + 1 + C$ $14 = C$ $y^2 = x^3 + x + 14$ $y = \sqrt{x^3+x+14}$ is branch with point $(1, 4)$ $f(x) = \sqrt{x^3+x+14}$	**5** { 1: separates variables 1: antiderivative of dy term 1: antiderivative of dx term 1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions 1: solves for y 0/1 if solving a linear equation in y 0/1 if no constant of integration Note: max 0/5 if no separation of variables Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation
(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$	**1**: answer, from student's solution to the given differential equation in (c)

1997 AB6/BC6
Solution

(a) $\frac{dv}{dt} = -2v - 32 = -2(v+16)$

$\frac{dv}{v+16} = -2dt$

$\ln\|v+16\| = -2t + A$

$\|v+16\| = e^{-2t+A} = e^A e^{-2t}$

$v + 16 = Ce^{-2t}$

$-50 + 16 = Ce^0;\ C = -34$

$v = -34e^{-2t} - 16$

(b) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(-34e^{-2t} - 16\right) = -16$

(c) $v(t) = -34e^{-2t} - 16 = -20$

$e^{-2t} = \frac{2}{17};\ t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$

1980 BC5
Solution

(a) $xy' + y = 0$

Solution using separation of variables	Solution using integrating factor
$\frac{y'}{y} = \frac{-1}{x} \quad \Rightarrow \quad \ln y = -\ln x + C$	$y' + \frac{1}{x}y = 0,\ \ P(x) = \frac{1}{x}$ $y = Ae^{-\int \frac{1}{x}dx}$

In either case, $y = Ae^{-\ln x}$ or $y = \frac{A}{x}$ or $xy = A$ or $\ln|y| = -\ln|x| + C$

(b) $xy' + y = 2x^2 y$

Solution using separation of variables	Solution using integrating factor
$\frac{y'}{y} = \frac{2x^2 - 1}{x} = 2x - \frac{1}{x}$ $\ln y = x^2 - \ln x + C$	$y' + \left(\frac{1 - 2x^2}{x}\right)y = 0,\quad P(x) = \frac{1}{x} - 2x$ $y = Ae^{-\int\left(\frac{1}{x} - 2x\right)dx}$

In either case, $y = \frac{Ae^{x^2}}{x}$ or $y = Ae^{x^2 - \ln x}$ or $\ln|y| = x^2 - \ln|x| + C$

(c) $e^2 = Ae$

$e = A$

Therefore $y = \frac{e \cdot e^{x^2}}{x} = \frac{e^{x^2+1}}{x}$ or $\ln y = x^2 - \ln x + 1$

1993 AB 6
Solution

(a) $P'(t) = k\left(800 - P(t)\right)$

$\frac{dP}{800 - P} = k\,dt$

$-\ln|800 - P| = kt + C_0$

$|800 - P| = C_1 e^{-kt}$

$800 - 500 = C_1 e^0$

$C_1 = 300$

Therefore $P(t) = 800 - 300e^{-kt}$

(b) $P(2) = 700 = 800 - 300e^{-2k}$

$k = \frac{\ln 3}{2} \approx 0.549$

(c) $\lim_{t\to\infty}\left(800 - 300e^{-\frac{\ln 3}{2}t}\right) = 800$