5. $y(2t_1) = \left(v_0 + \dfrac{mg}{p}\right)\dfrac{m}{p}\left(1 - e^{-2pt_1/m}\right) - \dfrac{mg}{p} \cdot 2t_1$

$= \left(\dfrac{pv_0 + mg}{p}\right)\dfrac{m}{p}\left[1 - \left(e^{pt_1/m}\right)^{-2}\right] - \dfrac{mg}{p} \cdot 2\dfrac{m}{p} \ln\left(\dfrac{pv_0 + mg}{mg}\right)$

Substituting $x = e^{pt_1/m} = \dfrac{pv_0}{mg} + 1 = \dfrac{pv_0 + mg}{mg}$ (from Problem 3), we get

$y(2t_1) = \left(x \cdot \dfrac{mg}{p}\right)\dfrac{m}{p}(1 - x^{-2}) - \dfrac{m^2 g}{p^2} \cdot 2\ln x = \dfrac{m^2 g}{p^2}\left(x - \dfrac{1}{x} - 2\ln x\right)$. Now $p > 0, m > 0, t_1 > 0 \;\Rightarrow$

$x = e^{pt_1/m} > e^0 = 1$. $f(x) = x - \dfrac{1}{x} - 2\ln x \;\Rightarrow\; f'(x) = 1 + \dfrac{1}{x^2} - \dfrac{2}{x} = \dfrac{x^2 - 2x + 1}{x^2} = \dfrac{(x-1)^2}{x^2} > 0$ for

$x > 1 \;\Rightarrow\; f(x)$ is increasing for $x > 1$. Since $f(1) = 0$, it follows that $f(x) > 0$ for every $x > 1$. Therefore,

$y(2t_1) = \dfrac{m^2 g}{p^2} f(x)$ is positive, which means that the ball has not yet reached the ground at time $2t_1$. This tells us that the time spent going up is always less than the time spent coming down, so *ascent is faster.*

9.4 Exponential Growth and Decay

1. The relative growth rate is $\dfrac{1}{P}\dfrac{dP}{dt} = 0.7944$, so $\dfrac{dP}{dt} = 0.7944P$ and, by Theorem 2,
$P(t) = P(0)e^{0.7944t} = 2e^{0.7944t}$. Thus, $P(6) = 2e^{0.7944(6)} \approx 234.99$ or about 235 members.

2. (a) By Theorem 2, $P(t) = P(0)e^{kt} = 60e^{kt}$. In 20 minutes ($\frac{1}{3}$ hour), there are 120 cells, so
$P(\frac{1}{3}) = 60e^{k/3} = 120 \;\Rightarrow\; e^{k/3} = 2 \;\Rightarrow\; k/3 = \ln 2 \;\Rightarrow\; k = 3\ln 2 = \ln(2^3) = \ln 8$.

(b) $P(t) = 60e^{(\ln 8)t} = 60 \cdot 8^t$

(c) $P(8) = 60 \cdot 8^8 = 60 \cdot 2^{24} = 1{,}006{,}632{,}960$

(d) $dP/dt = kP \;\Rightarrow\; P'(8) = kP(8) = (\ln 8)P(8) \approx 2.093$ billion cells/h

(e) $P(t) = 20{,}000 \;\Rightarrow\; 60 \cdot 8^t = 20{,}000 \;\Rightarrow\; 8^t = 1000/3 \;\Rightarrow\; t\ln 8 = \ln(1000/3) \;\Rightarrow$
$t = \dfrac{\ln(1000/3)}{\ln 8} \approx 2.79$ h

3. (a) By Theorem 2, $y(t) = y(0)e^{kt} = 500e^{kt}$. Now $y(3) = 500e^{k(3)} = 8000 \;\Rightarrow\; e^{3k} = \frac{8000}{500} \;\Rightarrow$
$3k = \ln 16 \;\Rightarrow\; k = (\ln 16)/3$. So $y(t) = 500e^{(\ln 16)t/3} = 500 \cdot 16^{t/3}$

(b) $y(4) = 500 \cdot 16^{4/3} \approx 20{,}159$

(c) $dy/dt = ky \;\Rightarrow\; y'(4) = ky(4) = \frac{1}{3}\ln 16\left(500 \cdot 16^{4/3}\right)$ [from part (a)] $\approx 18{,}631$ cells/h

(d) $y(t) = 500 \cdot 16^{t/3} = 30{,}000 \;\Rightarrow\; 16^{t/3} = 60 \;\Rightarrow\; \frac{1}{3}t\ln 16 = \ln 60 \;\Rightarrow\; t = 3(\ln 60)/(\ln 16) \approx 4.4$ h

4. (a) $y(t) = y(0)e^{kt} \;\Rightarrow\; y(2) = y(0)e^{2k} = 600$, $y(8) = y(0)e^{8k} = 75{,}000$. Dividing these equations, we get
$e^{8k}/e^{2k} = 75{,}000/600 \;\Rightarrow\; e^{6k} = 125 \;\Rightarrow\; 6k = \ln 125 = \ln 5^3 = 3\ln 5 \;\Rightarrow\; k = \frac{3}{6}\ln 5 = \frac{1}{2}\ln 5$.
Thus, $y(0) = 600/e^{2k} = 600/e^{\ln 5} = \frac{600}{5} = 120$.

(b) $y(t) = y(0)e^{kt} = 120e^{(\ln 5)t/2}$ or $y = 120 \cdot 5^{t/2}$

(c) $y(5) = 120 \cdot 5^{5/2} = 120 \cdot 25\sqrt{5} = 3000\sqrt{5} \approx 6708$ bacteria.

(d) $y(t) = 120 \cdot 5^{t/2} \Rightarrow y'(t) = 120 \cdot 5^{t/2} \cdot \ln 5 \cdot \frac{1}{2} = 60 \cdot \ln 5 \cdot 5^{t/2}$.
$y'(5) = 60 \cdot \ln 5 \cdot 5^{5/2} = 60 \cdot \ln 5 \cdot 25\sqrt{5} \approx 5398$ bacteria/hour.

(e) $y(t) = 200{,}000 \Leftrightarrow 120e^{(\ln 5)t/2} = 200{,}000 \Leftrightarrow e^{(\ln 5)t/2} = \frac{5000}{3} \Leftrightarrow (\ln 5)t/2 = \ln \frac{5000}{3} \Leftrightarrow$
$t = \left(2 \ln \frac{5000}{3}\right) / \ln 5 \approx 9.2$ h.

5. (a) Let the population (in millions) in the year t be $P(t)$. Since the initial time is the year 1750, we substitute $t - 1750$ for t in Theorem 2, so the exponential model gives $P(t) = P(1750)e^{k(t-1750)}$. Then
$P(1800) = 980 = 790e^{k(1800-1750)} \Rightarrow \frac{980}{790} = e^{k(50)} \Rightarrow \ln \frac{980}{790} = 50k \Rightarrow$
$k = \frac{1}{50} \ln \frac{980}{790} \approx 0.0043104$. So with this model, we have $P(1900) = 790e^{k(1900-1750)} \approx 1508$ million, and
$P(1950) = 790e^{k(1950-1750)} \approx 1871$ million. Both of these estimates are much too low.

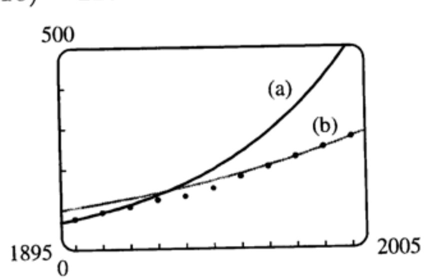
(b) In this case, the exponential model gives $P(t) = P(1850)e^{k(t-1850)} \Rightarrow$
$P(1900) = 1650 = 1260e^{k(1900-1850)} \Rightarrow \ln \frac{1650}{1260} = k(50) \Rightarrow k = \frac{1}{50} \ln \frac{1650}{1260} \approx 0.005393$. So with this model, we estimate $P(1950) = 1260e^{k(1950-1850)} \approx 2161$ million. This is still too low, but closer than the estimate of $P(1950)$ in part (a).

(c) The exponential model gives $P(t) = P(1900)e^{k(t-1900)} \Rightarrow P(1950) = 2560 = 1650e^{k(1950-1900)} \Rightarrow$
$\ln \frac{2560}{1650} = k(50) \Rightarrow k = \frac{1}{50} \ln \frac{2560}{1650} \approx 0.008785$. With this model, we estimate
$P(2000) = 1650e^{k(2000-1900)} \approx 3972$ million. This is much too low. The discrepancy is explained by the fact that the world birth rate (average yearly number of births per person) is about the same as always, whereas the mortality rate (especially the infant mortality rate) is much lower, owing mostly to advances in medical science and to the wars in the first part of the twentieth century. The exponential model assumes, among other things, that the birth and mortality rates will remain constant.

6. (a) Let $P(t)$ be the population (in millions) in the year t. Since the initial time is the year 1900, we substitute $t - 1900$ for t in Theorem 2, and find that the exponential model gives $P(t) = P(1900)e^{k(t-1900)} \Rightarrow$
$P(1910) = 92 = 76e^{k(1910-1900)} \Rightarrow k = \frac{1}{10} \ln \frac{92}{76} \approx 0.0191$. With this model, we estimate
$P(2000) = 76e^{k(2000-1900)} \approx 514$ million. This estimate is much too high. The discrepancy is explained by the fact that, between the years 1900 and 1910, an enormous number of immigrants (compared to the total population) came to the United States. Since that time, immigration (as a proportion of total population) has been much lower. Also, the birth rate in the United States has declined since the turn of the century. So our calculation of the constant k was based partly on factors which no longer exist.

(b) Substituting $t - 1980$ for t in Theorem 2, we find that the exponential model gives $P(t) = P(1980)e^{k(t-1980)}$
$\Rightarrow P(1990) = 250 = 227e^{k(1990-1980)} \Rightarrow k = \frac{1}{10} \ln \frac{250}{227} \approx 0.00965$. With this model, we estimate
$P(2000) = 227e^{k(2000-1980)} \approx 275.3$ million. This is quite accurate. The further estimates are
$P(2010) = 227e^{30k} \approx 303$ million and $P(2020) = 227e^{40k} \approx 334$ million.

(c)



The model in part (a) is quite inaccurate after 1910 (off by 5 million in 1920 and 12 million in 1930). The model in part (b) is more accurate (which is not surprising, since it is based on more recent information).

7. (a) If $y = [N_2O_5]$ then by Theorem 2, $\frac{dy}{dt} = -0.0005y \Rightarrow y(t) = y(0)e^{-0.0005t} = Ce^{-0.0005t}$.

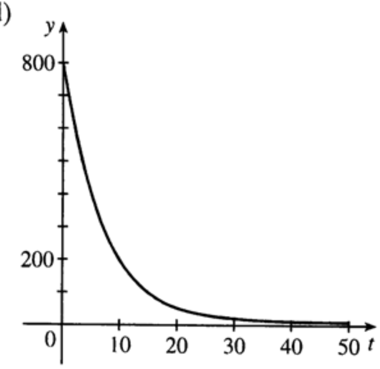
(b) $y(t) = Ce^{-0.0005t} = 0.9C \Rightarrow e^{-0.0005t} = 0.9 \Rightarrow -0.0005t = \ln 0.9 \Rightarrow$
$t = -2000 \ln 0.9 \approx 211$ s

8. (a) The mass remaining after t days is
$y(t) = y(0)e^{kt} = 800e^{kt}$. Since the half-life is 5.0 days,
$y(5) = 800e^{5k} = 400 \Rightarrow e^{5k} = \frac{1}{2} \Rightarrow$
$5k = \ln \frac{1}{2} \Rightarrow k = -(\ln 2)/5$, so
$y(t) = 800e^{-(\ln 2)t/5} = 800 \cdot 2^{-t/5}$.

(b) $y(30) = 800 \cdot 2^{-30/5} = 12.5$ mg

(c) $800e^{-(\ln 2)t/5} = 1 \Leftrightarrow -(\ln 2)\frac{t}{5} = \ln \frac{1}{800} = -\ln 800$
$\Leftrightarrow t = 5\frac{\ln 800}{\ln 2} \approx 48$ days

(d)



9. (a) If $y(t)$ is the mass (in mg) remaining after t years, then $y(t) = y(0)e^{kt} = 100e^{kt}$. $y(30) = 100e^{30k} = \frac{1}{2}(100)$
$\Rightarrow e^{30k} = \frac{1}{2} \Rightarrow k = -(\ln 2)/30 \Rightarrow y(t) = 100e^{-(\ln 2)t/30} = 100 \cdot 2^{-t/30}$

(b) $y(100) = 100 \cdot 2^{-100/30} \approx 9.92$ mg

(c) $100e^{-(\ln 2)t/30} = 1 \Rightarrow -(\ln 2)t/30 = \ln \frac{1}{100} \Rightarrow t = -30\frac{\ln 0.01}{\ln 2} \approx 199.3$ years

10. (a) If $y(t)$ is the mass after t days and $y(0) = A$, then $y(t) = Ae^{kt}$. $y(3) = Ae^{3k} = 0.58A \Rightarrow$
$e^{3k} = 0.58 \Rightarrow 3k = \ln 0.58 \Rightarrow k = \frac{1}{3}\ln 0.58$. Then $Ae^{(\ln 0.58)t/3} = \frac{1}{2}A \Leftrightarrow$
$\ln e^{(\ln 0.58)t/3} = \ln \frac{1}{2} \Leftrightarrow \frac{(\ln 0.58)t}{3} = \ln \frac{1}{2}$, so the half-life is $t = -\frac{3 \ln 2}{\ln 0.58} \approx 3.82$ days.

(b) $Ae^{(\ln 0.58)t/3} = 0.10A \Leftrightarrow \frac{(\ln 0.58)t}{3} = \ln \frac{1}{10} \Leftrightarrow t = -\frac{3 \ln 10}{\ln 0.58} \approx 12.68$ days

11. Let $y(t)$ be the level of radioactivity. Thus, $y(t) = y(0)e^{-kt}$ and k is determined by using the half-life:
$y(5730) = \frac{1}{2}y(0) \Rightarrow y(0)e^{-k(5730)} = \frac{1}{2}y(0) \Rightarrow e^{-5730k} = \frac{1}{2} \Rightarrow$
$-5730k = \ln \frac{1}{2} \Rightarrow k = -\frac{\ln \frac{1}{2}}{5730} = \frac{\ln 2}{5730}$. If 74% of the ^{14}C remains, then we know that $y(t) = 0.74y(0)$
$\Rightarrow 0.74 = e^{-t(\ln 2)/5730} \Rightarrow \ln 0.74 = -\frac{t \ln 2}{5730} \Rightarrow t = -\frac{5730(\ln 0.74)}{\ln 2} \approx 2489 \approx 2500$ years.

12. From the information given, we know that $\frac{dy}{dx} = 2y \Rightarrow y = Ce^{2x}$ by Theorem 2. To calculate C we use the point $(0, 5)$: $5 = Ce^{2(0)} \Rightarrow C = 5$. Thus, the equation of the curve is $y = 5e^{2x}$.

13. (a) Using Newton's Law of Cooling, $\frac{dT}{dt} = k(T - T_s)$, we have $\frac{dT}{dt} = k(T - 75)$.
Now let $y = T - 75$, so $y(0) = T(0) - 75 = 185 - 75 = 110$, so y is a solution of the initial-value problem $dy/dt = ky$ with $y(0) = 110$ and by Theorem 2 we have $y(t) = y(0)e^{kt} = 110e^{kt}$.
$y(30) = 110e^{30k} = 150 - 75 \Rightarrow e^{30k} = \frac{75}{110} = \frac{15}{22} \Rightarrow k = \frac{1}{30}\ln \frac{15}{22}$,
so $y(t) = 110e^{\frac{1}{30}t\ln(\frac{15}{22})}$ and $y(45) = 110e^{\frac{45}{30}\ln(\frac{15}{22})} \approx 62°F$. Thus, $T(45) \approx 62 + 75 = 137°F$.

(b) $T(t) = 100 \Rightarrow y(t) = 25$. $y(t) = 110e^{\frac{1}{30}t\ln(\frac{15}{22})} = 25 \Rightarrow e^{\frac{1}{30}t\ln(\frac{15}{22})} = \frac{25}{110} \Rightarrow$
$\frac{1}{30}t \ln \frac{15}{22} = \ln \frac{25}{110} \Rightarrow t = \frac{30 \ln \frac{25}{110}}{\ln \frac{15}{22}} \approx 116$ min.

14. (a) Let $T(t)$ = temperature after t minutes. Newton's Law of Cooling implies that $\dfrac{dT}{dt} = k(T-5)$. Let $y(t) = T(t) - 5$. Then $\dfrac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt} = 15e^{kt} \;\Rightarrow\; T(t) = 5 + 15e^{kt} \;\Rightarrow$ $T(1) = 5 + 15e^{k} = 12 \;\Rightarrow\; e^{k} = \frac{7}{15} \;\Rightarrow\; k = \ln\frac{7}{15}$, so $T(t) = 5 + 15e^{\ln(7/15)t}$ and $T(2) = 5 + 15e^{2\ln(7/15)} \approx 8.3°\text{C}$.

(b) $5 + 15e^{\ln(7/15)t} = 6$ when $e^{\ln(7/15)t} = \frac{1}{15} \;\Rightarrow\; \ln\left(\frac{7}{15}\right)t = \ln\frac{1}{15} \;\Rightarrow\; t = \dfrac{\ln\frac{1}{15}}{\ln\frac{7}{15}} \approx 3.6$ min.

15. $\dfrac{dT}{dt} = k(T-20)$. Letting $y = T - 20$, we get $\dfrac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$.
$y(0) = T(0) - 20 = 5 - 20 = -15$, so $y(25) = y(0)e^{25k} = -15e^{25k}$, and
$y(25) = T(25) - 20 = 10 - 20 = -10$, so $-15e^{25k} = -10 \;\Rightarrow\; e^{25k} = \frac{2}{3}$. Thus, $25k = \ln\left(\frac{2}{3}\right)$ and
$k = \frac{1}{25}\ln\left(\frac{2}{3}\right)$, so $y(t) = y(0)e^{kt} = -15e^{(1/25)\ln(2/3)t}$. More simply, $e^{25k} = \frac{2}{3} \;\Rightarrow\; e^{k} = \left(\frac{2}{3}\right)^{1/25} \;\Rightarrow$
$e^{kt} = \left(\frac{2}{3}\right)^{t/25} \;\Rightarrow\; y(t) = -15\cdot\left(\frac{2}{3}\right)^{t/25}$.

(a) $T(50) = 20 + y(50) = 20 - 15\cdot\left(\frac{2}{3}\right)^{50/25} = 20 - 15\cdot\left(\frac{2}{3}\right)^{2} = 20 - \frac{20}{3} = 13.\bar{3}\,°\text{C}$

(b) $15 = T(t) = 20 + y(t) = 20 - 15\cdot\left(\frac{2}{3}\right)^{t/25} \;\Rightarrow\; 15\cdot\left(\frac{2}{3}\right)^{t/25} = 5 \;\Rightarrow\; \left(\frac{2}{3}\right)^{t/25} = \frac{1}{3} \;\Rightarrow$
$(t/25)\ln\left(\frac{2}{3}\right) = \ln\left(\frac{1}{3}\right) \;\Rightarrow\; t = 25\ln\left(\frac{1}{3}\right)/\ln\left(\frac{2}{3}\right) \approx 67.74$ min.

16. $\dfrac{dT}{dt} = k(T-20)$. Let $y = T - 20$. Then $\dfrac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $y(0) = T(0) - 20 = 95 - 20 = 75$, so $y(t) = 75e^{kt}$. When $T(t) = 70$, $\dfrac{dT}{dt} = -1°\text{C/min}$. Equivalently, $\dfrac{dy}{dt} = -1$ when $y(t) = 50$. Thus,
$-1 = \dfrac{dy}{dt} = ky(t) = 50k$ and $50 = y(t) = 75e^{kt}$. The first relation implies $k = -1/50$, so the second relation says $50 = 75e^{-t/50}$. Thus, $e^{-t/50} = \frac{2}{3} \;\Rightarrow\; -t/50 = \ln\left(\frac{2}{3}\right) \;\Rightarrow\; t = -50\ln\left(\frac{2}{3}\right) \approx 20.27$ min.

17. (a) Let $P(h)$ be the pressure at altitude h. Then $dP/dh = kP \;\Rightarrow\; P(h) = P(0)e^{kh} = 101.3e^{kh}$.
$P(1000) = 101.3e^{1000k} = 87.14 \;\Rightarrow\; 1000k = \ln\left(\frac{87.14}{101.3}\right) \;\Rightarrow$
$k = \frac{1}{1000}\ln\left(\frac{87.14}{101.3}\right) \;\Rightarrow\; P(h) = 101.3\,e^{\frac{1}{1000}h\ln\left(\frac{87.14}{101.3}\right)}$, so $P(3000) = 101.3e^{3\ln\left(\frac{87.14}{101.3}\right)} \approx 64.5$ kPa.

(b) $P(6187) = 101.3\,e^{\frac{6187}{1000}\ln\left(\frac{87.14}{101.3}\right)} \approx 39.9$ kPa

18. (a) Using $A = A_0\left(1 + \dfrac{r}{n}\right)^{nt}$ with $A_0 = 500$, $r = 0.14$, and $t = 2$, we have:

(i) Annually: $n = 1$; $A = 500\left(1 + \frac{0.14}{1}\right)^{1\cdot 2} = \$649.80$

(ii) Quarterly: $n = 4$; $A = 500\left(1 + \frac{0.14}{4}\right)^{4\cdot 2} = \$658.40$

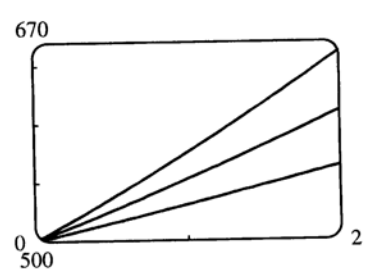
(iii) Monthly: $n = 12$; $A = 500\left(1 + \frac{0.14}{12}\right)^{12\cdot 2} = \$660.49$

(iv) Daily: $n = 365$; $A = 500\left(1 + \frac{0.14}{365}\right)^{365\cdot 2} = \$661.53$

(v) Hourly: $n = 365\cdot 24$; $A = 500\left(1 + \frac{0.14}{365\cdot 24}\right)^{365\cdot 24\cdot 2} = \$661.56$

(vi) Continuously: $A = 500e^{(0.14)2} = \$661.56$

(b)



$A_{0.14}(2) = \$661.56$,
$A_{0.10}(2) = \$610.70$, and
$A_{0.06}(2) = \$563.75$.

19. (a) Using $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$ with $A_0 = 3000$, $r = 0.05$, and $t = 5$, we have:

(i) Annually: $n = 1$; $A = 3000\left(1 + \frac{0.05}{1}\right)^{1\cdot 5} = \$3828.84$

(ii) Semiannually: $n = 2$; $A = 3000\left(1 + \frac{0.05}{2}\right)^{2\cdot 5} = \$3840.25$

(iii) Monthly: $n = 12$; $A = 3000\left(1 + \frac{0.05}{12}\right)^{12\cdot 5} = \$3850.08$

(iv) Weekly: $n = 52$; $A = 3000\left(1 + \frac{0.05}{52}\right)^{52\cdot 5} = \$3851.61$

(v) Daily: $n = 365$; $A = 3000\left(1 + \frac{0.05}{365}\right)^{365\cdot 5} = \$3852.01$

(vi) Continuously: $A = 3000e^{(0.05)5} = \$3852.08$

(b) $dA/dt = 0.05A$ and $A(0) = 3000$.

20. (a) $A_0e^{0.06t} = 2A_0 \;\Leftrightarrow\; e^{0.06t} = 2 \;\Leftrightarrow\; 0.06t = \ln 2 \;\Leftrightarrow\; t = \frac{50}{3}\ln 2 \approx 11.55$, so the investment will double in about 11.55 years.

(b) The annual interest rate in $A = A_0(1 + r)^t$ is r. From part (a), we have $A = A_0e^{0.06t}$. These amounts must be equal, so $(1 + r)^t = e^{0.06t} \;\Rightarrow\; 1 + r = e^{0.06} \;\Rightarrow\; r = e^{0.06} - 1 \approx 0.0618 = 6.18\%$, which is the equivalent annual interest rate.

21. (a) $\dfrac{dP}{dt} = kP - m = k\left(P - \dfrac{m}{k}\right)$. Let $y = P - \dfrac{m}{k}$, so $\dfrac{dy}{dt} = \dfrac{dP}{dt}$ and the differential equation becomes $\dfrac{dy}{dt} = ky$. The solution is $y = y_0e^{kt} \;\Rightarrow\; P - \dfrac{m}{k} = \left(P_0 - \dfrac{m}{k}\right)e^{kt} \;\Rightarrow\; P(t) = \dfrac{m}{k} + \left(P_0 - \dfrac{m}{k}\right)e^{kt}$.

(b) Since $k > 0$, there will be an exponential expansion $\;\Leftrightarrow\; P_0 - \dfrac{m}{k} > 0 \;\Leftrightarrow\; m < kP_0$.

(c) The population will be constant if $P_0 - \dfrac{m}{k} = 0 \;\Leftrightarrow\; m = kP_0$. It will decline if $P_0 - \dfrac{m}{k} < 0 \;\Leftrightarrow\; m > kP_0$.

(d) $P_0 = 8{,}000{,}000$, $k = \alpha - \beta = 0.016$, $m = 210{,}000 \;\Rightarrow\; m > kP_0\ (= 128{,}000)$, so by part (c), the population was declining.

22. (a) $\dfrac{dy}{dt} = ky^{1+c} \;\Rightarrow\; y^{-1-c}\,dy = k\,dt \;\Rightarrow\; \dfrac{y^{-c}}{-c} = kt + C$. Since $y(0) = y_0$, we have $C = \dfrac{y_0^{-c}}{-c}$. Thus, $\dfrac{y^{-c}}{-c} = kt + \dfrac{y_0^{-c}}{-c}$, or $y^{-c} = y_0^{-c} - ckt$. So $y^c = \dfrac{1}{y_0^{-c} - ckt} = \dfrac{y_0^c}{1 - cy_0^ckt}$ and $y(t) = \dfrac{y_0}{(1 - cy_0^ckt)^{1/c}}$.

(b) $y(t) \to \infty$ as $1 - cy_0^ckt \to 0$, that is, as $t \to \dfrac{1}{cy_0^ck}$. Define $T = \dfrac{1}{cy_0^ck}$. Then $\lim\limits_{t\to T^-} y(t) = \infty$.

(c) According to the data given, we have $c = 0.01$, $y(0) = 2$, and $y(3) = 16$, where the time t is given in months. Thus, $y_0 = 2$ and $16 = y(3) = \dfrac{y_0}{(1 - cy_0^ck\cdot 3)^{1/c}}$. Since $T = \dfrac{1}{cy_0^ck}$, we will solve for cy_0^ck.

$16 = \dfrac{2}{(1 - 3cy_0^ck)^{100}} \;\Rightarrow\; 1 - 3cy_0^ck = \left(\frac{1}{8}\right)^{0.01} = 8^{-0.01} \;\Rightarrow\; cy_0^ck = \frac{1}{3}\left(1 - 8^{-0.01}\right)$. Thus, doomsday occurs when $t = T = \dfrac{1}{cy_0^ck} = \dfrac{3}{1 - 8^{-0.01}} \approx 145.77$ months or 12.15 years.