**AP Calculus AB** **Name:** ____________________

**Integrals and FTC MC Review**

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**1.** $\int_0^1 \sqrt{x^2-2x+1}\,dx$ is $= \int_0^1 \sqrt{(x-1)^2}\,dx = \int_0^1 |x-1|\,dx = -\int_0^1 (x-1)\,dx = -\left(\frac{1}{2}x^2-x\right)\Big|_0^1$

$= -\left(\frac{1}{2}\cdot 1^2 - 1\right) = 1/2$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. none of the above

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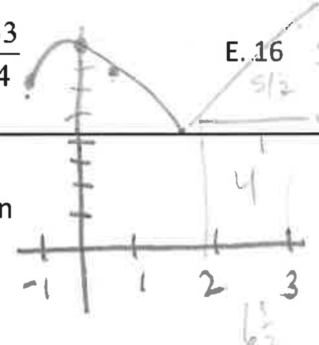
**2.** What is the average (mean) value of $3t^3-t^2$ over the interval $-1 \le t \le 2$? $\frac{1}{3}\left[\frac{3}{4}(2)^4-\frac{1}{3}(2)^3\right] - \frac{1}{3}\left[\frac{3}{4}(-1)^4-\frac{1}{3}(-1)^3\right]$

$f_{ave} = \frac{1}{2--1}\int_{-1}^{2}(3t^3-t^2)\,dt = \frac{1}{3}\left[\frac{3}{4}t^4-\frac{1}{3}t^3\right]_{-1}^{2}$ $= 4-\frac{8}{9}-\frac{1}{4}-\frac{1}{9} = \frac{11}{4}$

A. $\frac{11}{4}$ B. $\frac{7}{2}$ C. 8 D. $\frac{33}{4}$ E. 16



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**3.** If $\begin{cases} f(x) = 8-x^2 & \textit{for } -2 \le x \le 2 \\ f(x) = x^2 & \textit{elsewhere} \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

$\int_{-1}^{2}(8-x^2)\,dx + \int_2^3 x^2\,dx$

$8x-\frac{1}{3}x^3\Big|_{-1}^{2} + \frac{1}{3}x^3\Big|_2^3$

A. 0 and 8 B. 8 and 16 C. 16 and 24 D. 24 and 32 E. 32 and 40

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**4.** If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$ $e^{-x^2}$

A. $2xe^{-x^2}$ B. $-2xe^{-x^2}$ C. $\frac{e^{-x^2+1}}{-x^2+1} - e$ D. $e^{-x^2}-1$ E. $e^{-x^2}$

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**5.** If $f(x) = \int_0^x \frac{1}{\sqrt{t^3+2}}\,dt$, which of the following is FALSE?

A. $f(0)=0$ true $\int_0^0 \frac{1}{t^3+2}\,dt = 0$

B. B. $f$ is continuous at $x$ for all $x \ge 0$ true

C. C. $f(1) > 0$ true

D. D. $f'(1) = \frac{1}{\sqrt{3}}$ $f'(x) = \frac{1}{\sqrt{x^3+2}} = \frac{1}{\sqrt{3}}$ true

E. E. $f(-1) > 0$ $f(-1) = \int_0^{-1} \frac{1}{\sqrt{t^3+2}}\,dt$

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$= -\int_{-1}^{0} \frac{1}{\sqrt{t^3+2}}\,dt = -.765$

6. If *F* and *f* are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is $= F(x)\Big|_a^b$

A. $F'(a) - F'(b)$

B. $F'(b) - F'(a)$

C. $F(a) - F(b)$

$= F(b) - F(a)$

D. $F(b) - F(a)$

E. none of the above

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7. $\int (x^3 - 3x)\,dx =$ $\frac{1}{4}x^4 - \frac{3}{2}x^2 + C$

A. $3x^2 - 3 + C$

B. $4x^4 - 6x^2 + C$

C. $\frac{x^4}{3} - 3x^2 + C$

D. $\frac{x^4}{4} - 3x + C$

E. $\frac{x^4}{4} - \frac{3x^2}{2} + C$

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8. $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx =$ $\sec x \tan x \Big|_0^{\pi/4}$

A. $\frac{\pi}{4} - 1$

B. $1 - \frac{\pi}{4}$

C. $\frac{1}{3}$

D. $\sqrt{2} - 1$

E. $\frac{\pi}{4} + 1$

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9. $\int \frac{5}{1+x^2}\,dx =$ $5\int \frac{1}{1+x^2}\,dx = 5\tan^{-1}x + C$

A. $\frac{-10x}{(1+x^2)^2} + C$

B. $\frac{5}{2x}\ln(1+x^2) + C$

C. $5x - \frac{5}{x} + C$

D. $5\arctan x + C$

E. $5\ln(1+x^2) + C$

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10. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is $f_{ave} = \frac{1}{2-0}\int_0^2 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big|_0^2$

$= \frac{1}{2}\cdot\frac{2}{3}(2)^{3/2} = \frac{1}{3}\sqrt{8}$

$= \frac{2}{3}\sqrt{2}$

A. $\frac{\sqrt{2}}{3}$

B. $\frac{\sqrt{2}}{2}$

C. $\frac{2\sqrt{2}}{3}$

D. 1

E. $\frac{4\sqrt{2}}{3}$

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11. $\int_1^2 x^{-3}\,dx =$ $-\frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{2}\left[2^{-2} - 1^{-2}\right] = -\frac{1}{2}\left[\frac{1}{4} - 1\right] = -\frac{1}{2}\cdot -\frac{3}{4} = \frac{3}{8}$

A. $-\frac{7}{8}$

B. $-\frac{3}{4}$

C. $\frac{15}{64}$

D. $\frac{3}{8}$

E. $\frac{15}{16}$

**12.** Which of the following is equal to $\ln 4$ ?  $e^4 - e^1$  $\ln|x|\Big|_1^4 = \ln 4 - \ln 1 = \ln 4$

A. $\ln 3 + \ln 1$  B. $\dfrac{\ln 8}{\ln 2}$  C. $\int_1^4 e^t\,dt$  D. $\int_1^4 \ln x\,dx$  $\frac{1}{x}\Big|_1^4$  E. $\int_1^4 \frac{1}{t}\,dt$

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**13.** $\int_1^2 \dfrac{x^2-1}{x+1}\,dx = \int_1^2 \frac{(x-1)(x+1)}{\cancel{x+1}}\,dx = \int_1^2 (x-1)\,dx = \frac{1}{2}x^2 - x\Big|_1^2 = \left[\frac{1}{2}(2)^2 - 2\right] - \left(\frac{1}{2}(1^2) - 1\right) = 0 - -\frac{1}{2} = \frac{1}{2}$

A. $\dfrac{1}{2}$  B. 1  C. 2  D. $\dfrac{5}{2}$  E. ln 3

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**14.** If $\int_{-2}^{2}\left(x^7 + k\right)dx = 16$ , then k =

$\frac{1}{8}x^8 + kx\Big|_{-2}^{2} = 16$

$\frac{1}{8}(2)^8 + 2k - \frac{1}{8}(-2)^8 - k(-2) = 16$

$32 + 2k - 32 + 2k = 16$

$4k = 16$

$k = 4$

A. -12  B. -4  C. 0  D. 4  E. 12

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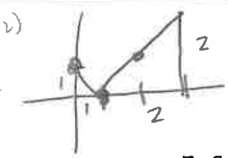
**15.** $\int_0^3 |x-1|\,dx = -\int_0^1 (x-1)\,dx + \int_1^3 (x-1)\,dx$

$-\left(\frac{1}{2}x^2 - x\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^3$

$\frac{1}{2} + \left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right) = 2 - -\frac{1}{2} = 5/2$

$\frac{1}{2}(1\cdot 1) + \frac{1}{2}(2)(2)$

$\frac{1}{2} + 2 = \frac{5}{2}$



A. 0  B. $\dfrac{3}{2}$  C. 2  D. $\dfrac{5}{2}$  E. 6

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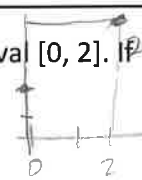
**16.** Let *f* be a continuous function on the closed interval [0, 2]. If $2 \le f(x) \le 4$ , then the greatest possible value of $\int_0^2 f(x)\,dx$ is



$2 \cdot 4 = 8$

A. 0  B. 2  C. 4  D. 8  E. 16

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**17.** $\int \sec^2 x\,dx =$ $\tan x + C$

A. $\tan x + C$  B. $\csc^2 x + C$  C. $\cos^2 x + C$

D. $\dfrac{\sec^3 x}{3} + C$  E. $2\sec^2 x \tan x + C$

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**18.** If $\int_0^k \left(2kx - x^2\right)dx = 18$ , then k =  $kx^2 - \frac{1}{3}x^3\Big|_0^k = 18$

$k \cdot k^2 - \frac{1}{3}k^3 = 18$

$k^3 - \frac{1}{3}k^3 = 18$

$2/3\,k^3 = 18$

$k^3 = 27$

$k = 3$

A. -9  B. -3  C. 3  D. 9  E. 18

**19.** $\int_1^4 |x-3|\,dx =$

$\frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 1\cdot 1 = 2+\frac{1}{2} = \frac{5}{2}$

A. $-\frac{3}{2}$ B. $\frac{3}{2}$ C. $\frac{5}{2}$ D. $\frac{9}{2}$ E. 5

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**20.** $\int_0^1 (3x-2)^2\,dx =$ $\int_0^1 (9x^2-12x+4)\,dx = 3x^3-6x^2+4x\Big|_0^1 = 3(1)^3-6(1)^2+4(1) = 3-6+4 = 1$

A. $-\frac{7}{3}$ B. $-\frac{7}{9}$ C. $\frac{1}{9}$ D. 1 E. 3

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**21.** If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$ , then $F'(x)=$ $\sqrt{1+(x^3)^2}\cdot 2x = 2x\sqrt{1+x^6}$

A. $2x\sqrt{1+x^6}$ B. $2x\sqrt{1+x^3}$ C. $\sqrt{1+x^6}$ D. $\sqrt{1+x^3}$ E. $\int_1^{x^2}\frac{3t^2}{2\sqrt{1+t^3}}dt$

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**22.** The average value of $\frac{1}{x}$ on the closed interval [1, 3] is

$\frac{1}{3-1}\int_1^3 \frac{1}{x}dx = \frac{1}{2}\ln|x|\Big|_1^3 = \frac{1}{2}(\ln 3-\ln 1)$

$\frac{1}{2}\ln 3 = \frac{\ln 3}{2}$

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{\ln 2}{2}$ D. $\frac{\ln 3}{2}$ E. ln3

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**23.** $\int (x^2+1)^2\,dx =$ $\int (x^4+2x^2+1)\,dx = \frac{1}{5}x^5+\frac{2}{3}x^3+x+C$

A. $\frac{(x^2+1)^3}{3}+C$ B. $\frac{(x^2+1)^3}{6x}+C$ C. $\left(\frac{x^3}{3}+x\right)^2+C$

D. $\frac{2x(x^2+1)^3}{3}+C$ E. $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

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**24.** $\int_1^{500}(13^x-11^x)\,dx+\int_2^{500}(11^x-13^x)\,dx =$ $\int^2 (13^x-11^x)\,dx$

$\boxed{\int_1^2 (13^x-11^x)\,dx} + \boxed{\int_2^{500}(13^x-11^x)\,dx + \int_2^{500}-(13^x-11^x)\,dx}$ cancel

Calc.

A. 0.000 B. 14.946 C. 34.415 D. 46.000 E. 136.364

**25.** If the second derivative of $f$ is given by $f''(x) = 2x - \cos x$ , which of the following could be $f(x)$ ?

$f'(x) = x^2 - \sin x + C$
$f(x) = \frac{1}{3}x^3 + \cos x + Cx$

A. $\frac{x^3}{3} + \cos x - x + 1$     B. $\frac{x^3}{3} - \cos x - x + 1$     C. $x^3 + \cos x - x + 1$

D. $x^2 - \sin x + 1$     E. $x^2 + \sin x + 1$

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**26.** If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$ , which of the following must be true?

I. $f(x) > g(x)$ $for$ $a \le x \le b$

II. $\int_a^b (f(x) + g(x))\,dx = 4$

III. $\int_a^b (f(x)g(x))\,dx = -5$

A. I only     B. II only     C. III only     D. II and III only     E. I, II, and III

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**27.** Which of the following is equal to $\int_0^{\pi} \sin x\,dx$ ? $= \cos x \Big|_0^{\pi} = \cos \pi - \cos 0$

$-1 - 1 = -2$

A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$     B. $\int_0^{\pi} \cos x\,dx$     C. $\int_{-\pi}^{0} \sin x\,dx$

D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx$     E. $\int_{\pi}^{2\pi} \sin x\,dx$

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**28.** If $\int_a^b f(x)dx = a + 2b$ , then $\int_a^b (f(x) + 5)\,dx =$    $\int_a^b f(x)\,dx + \int_a^b 5\,dx$

$a + 2b = 5x\Big|_a^b = 5b - 5a$

$a + 2b + 5b - 5a = 7b - 4a$

A. $a + 2b + 5$     B. $5b - 5a$     C. $7b - 4a$     D. $7b - 5a$     E. $7b - 6a$

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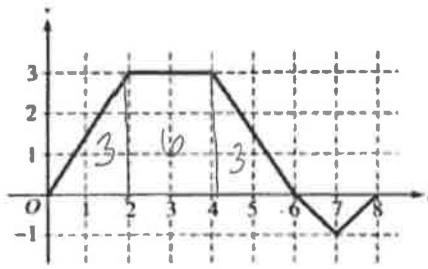
**29.** $\int_1^2 (4x^3 - 6x)\,dx =$    $x^4 - 3x^2 \Big|_1^2 = (2^4 - 3\cdot 2^2) - (1^4 - 3\cdot 1^2)$

$16 - 12 + 2 = 6$

A. 2     B. 4     C. 6     D. 36     E. 42




A bug begins to crawl up a vertical wire at time t = 0. The velocity $v$ of the bug at time $t$, interval, is given by the function whose graph is shown above.

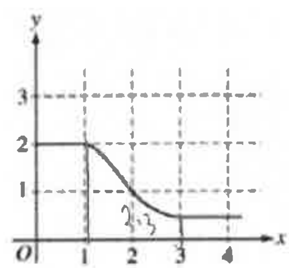
**30.** At what value of $t$ does the bug change direction? → when Velocity changes from pos to neg

A. 2 B. 4 C. 6 D. 7 E. 8

$\int_0^6 v(t)\,dt + -\int_6^8 v(t)\,dt = 12+1 = 13$

**31.** What is the total distance the bug traveled from $t = 0$ to $t = 8$?

A. 14 B. 13 C. 11 D. 8 E. 6




**32.** The graph of $f$ is shown in the figure above. If $\int_1^3 f(x)dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$ $\int_0^3 F'(x)\,dx = 2.3 + 2$

A. 0.3 B. 1.3 C. 3.3 D. 4.3 E. 5.3

**33.** $\int_0^x \sin t\,dt =$ $-\cos t \Big|_0^x = -\cos x + \cos 0 = -\cos x + 1 = 1 - \cos x$

A. $\sin x$ B. $-\cos x$ C. $\cos x$ D. $\cos x - 1$ E. $1 - \cos x$

**34.** $\int_0^1 \sqrt{x}(x+1)\,dx =$ $\int_0^1 x^{1/2}(x+1)\,dx = \int_0^1 (x^{3/2} + x^{1/2})\,dx = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{6+10}{15} = \frac{16}{15}$

A. 0 B. 1 C. $\frac{16}{15}$ D. $\frac{7}{5}$ E. 2

**35.** Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$ $= \frac{1}{2}(\sin x)^2$ $F'(x) = \sin x \cos x$

II. $F(x) = \dfrac{\cos^2 x}{2}$ $\frac{1}{2}(\cos x)^2$ $F'(x)\ \cos x(-\sin x) = -\sin x \cos x$

III. $F(x) = \dfrac{-\cos(2x)}{4}$ $-\frac{1}{4}\cos 2x$ $F'(x) = -\frac{1}{4}(-\sin 2x)\cdot 2 = \frac{1}{2}\sin 2x$
$= \frac{1}{2}\cdot(2\sin x \cos x)$
$= \sin x \cos x$
(Double & form)

A. I only B. II only C. III only D. I and III only E. II and III only

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**36.** $\int_1^e \left(\dfrac{x^2-1}{x}\right)dx =$ $\int_1^e \left(x - \frac{1}{x}\right)dx = \frac{1}{2}x^2 - \ln|x|\Big|_1^e = \left(\frac{1}{2}e^2 - \ln e\right) - \left(\frac{1}{2} - \ln|1|\right)$
$\frac{1}{2}e^2 - 1 - \frac{1}{2} - 0 = \frac{1}{2}e^2 - \frac{3}{2}$

A. $e - \dfrac{1}{e}$ B. $e^2 - e$ C. $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ D. $e^2 - 2$ E. $\dfrac{e^2}{2} - \dfrac{3}{2}$

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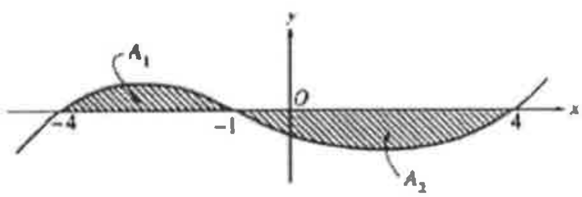


**37.** The graph of $y = f(x)$ is shown in the figure above. If $A_1$ and $A_2$ are positive numbers that represent the areas of the shaded regions, then in terms of $A_1$ and $A_2$, $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$ $=$
$A_1 + -A_2 - 2(-A_2) = A_1 - A_2 + 2A_2 =$

A. $A_1$ B. $A_1 - A_2$ C. $2A_1 - A_2$ D. $A_1 + A_2$ E. $A_1 + 2A_2$ $A_1 + A_2$

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$F'(x) = \sqrt{x^3+1}$

**38.** If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$
$\sqrt{2^3+1} = \sqrt{9} = 3$

A -3 B. -2 C. 2 D. 3 E. 18

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**39.** What are all values of $k$ for which $\int_{-3}^{k} x^2 dx = 0$? $\frac{1}{3}x^3\Big|_{-3}^{k} = 0$
$\frac{1}{3}k^3 - \frac{1}{3}(-3)^3 = 0$

A. -3 B. 0 C. 3 D. -3 and 3 E. -3, 0, and 3

$\frac{1}{3}k^3 + 9 = 0$

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$\frac{1}{3}k^3 = -9$
$k^3 = -27$
$k = -3$

**40.** If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

$\int_k^{\pi/2} \cos x \, dx = .1$ $\quad \sin x \Big|_k^{\pi/2} = .1$ $\quad \sin\frac{\pi}{2} - \sin k = .1$

$1 - \sin k = .1$

$-\sin k = -.9$

$\sin k = .9$

$k = 1.1120$

A. 1.471 B. 1.414 C. 1.277 D. 1.120 E. 0.436

**41.** If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx$

A. $2\int_3^5 g(x)dx + 7$

B. $2\int_3^5 g(x)dx + 14$

C. $2\int_3^5 g(x)dx + 28$

D. $\int_3^5 g(x)dx + 7$

E. $\int_3^5 g(x)dx + 14$

$\int_3^5 g(x) + 7 + g(x)\,dx$

$= 2\int_3^5 (g(x) + 7)\,dx$

$2\int_3^5 g(x)\,dx + 7x\Big|_3^5$

$7(5) - 7(3)$

$35 - 21$

$2\int_3^5 g(x)dx + 14$

**42.** $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

$\cos 2\pi x$

FTC pt II

A. 0 B. $\frac{1}{2\pi}\sin x$ C. $\frac{1}{2\pi}\cos(2\pi x)$

D. $\cos(2\pi x)$ E. $2\pi\cos(2\pi x)$

# Integrals and FTC MC Review

1. C
2. A
3. D
4. E
5. E
6. D
7. E
8. B
9. D
10. C
11. D
12. E
13. A
14. D
15. D
16. D
17. A
18. C
19. C
20. D
21. A
22. D
23. E
24. B
25. A
26. B
27. A
28. C
29. C
30. C
31. B
32. D
33. E
34. C
35. D
36. E
37. D
38. D
39. A
40. D
41. B
42. D