(1) $\int \frac{1}{\sqrt{9-x^2}} = \int \frac{1}{\sqrt{\left(\frac{9}{9}-\frac{x^2}{9}\right)9}}\, dx = \int \frac{1}{3\sqrt{1-\left(\frac{x}{3}\right)^2}}\, dx$

$u = \frac{x}{3}$

$du = \frac{1}{3} dx$

$dx = 3\, du$

$\int \frac{1}{\cancel{3}\sqrt{1-u^2}} \cdot \cancel{3}\, du = \int \frac{1}{\sqrt{1-u^2}}\, du$

$= \sin^{-1} u + c$

$= \sin^{-1}\left(\frac{x}{3}\right) + c$

(2) $\int \frac{1}{\sqrt{1-(2x)^2}}\, dx$

$u = 2x$

$du = 2\, dx$

$dx = \frac{du}{2}$

$= \int \frac{1}{\sqrt{1-u^2}} \frac{du}{2} = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}}\, du = \frac{1}{2} \sin^{-1}(u) + c$

$= \frac{1}{2} \sin^{-1}(2x) + c$

③ $$\int \frac{1}{17+x^2}\,dx = \int \frac{1}{\left(\frac{17}{17}+\frac{x^2}{17}\right)17}\,dx$$

$$= \frac{1}{17}\int \frac{1}{1+\left(\frac{1}{\sqrt{17}}x\right)^2}\,dx$$

$u = \frac{x}{\sqrt{17}}$

$du = \frac{1}{\sqrt{17}}\,dx$

$dx = \sqrt{17}\,du$

$$= \frac{1}{17}\int \frac{1}{1+u^2}\cdot\sqrt{17}\,du$$

$$= \frac{\sqrt{17}}{17}\int \frac{1}{1+u^2}\,du$$

$$= \frac{\sqrt{17}}{17}\tan^{-1}u + C$$

$$= \frac{\sqrt{17}}{17}\tan^{-1}\left(\frac{x}{\sqrt{17}}\right) + C$$

(4) $\int \frac{1}{9+3x^2}\,dx = \int \frac{1}{3(3+x^2)}\,dx$

$= \frac{1}{3}\int \frac{1}{\left(\left(\frac{3}{3}\right)+\frac{x^2}{3}\right)3} = \frac{1}{9}\int \frac{1}{1+\frac{1}{3}x^2} = \frac{1}{9}\int \frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2}$

$u = \frac{x}{\sqrt{3}}$

$du = \frac{1}{\sqrt{3}}\,dx$

$dx = \sqrt{3}\,du$

$= \frac{1}{9}\int \frac{1}{1+u^2}\,du \cdot \sqrt{3}$

$= \frac{\sqrt{3}}{9}\tan^{-1} u + C$

$= \frac{\sqrt{3}}{9}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(5) $\int_0^1 \frac{4}{\sqrt{4-x^2}}\,dx = \int_0^1 \frac{4}{\sqrt{\left(\frac{4}{4}+\frac{x^2}{9}\right)4}}\,dx$

$= \int_0^1 \frac{4}{2\sqrt{1+\left(\frac{x}{2}\right)^2}}\,dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}\,dx$

$dx = du \cdot 2$

$= \int \frac{2}{\sqrt{1+u^2}}\,du \cdot 2$

$= 4\int_0^{1/2} \frac{1}{\sqrt{1+u^2}}$ $\quad 4\sin^{-1} u \Big|_0^{1/2}$ $= 4\left[\sin^{-1}\tfrac{1}{2} - \sin^{-1} 0\right]$

$= 4\left[\frac{\pi}{6} - 0\right]$

$= \frac{4\pi}{6} = \frac{2\pi}{3}$

(6) $\int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9-4x^2}}\,dx = \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{\left(\frac{9}{9}+\frac{4x^2}{9}\right)9}} = \frac{1}{3}\int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{1+\left(\frac{2}{3}x\right)^2}}\,dx$

$u = \frac{2}{3}x$

$du = \frac{2}{3}\,dx$

$dx = \frac{3}{2}\,du$

$\frac{3}{2}\cdot\frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\int_0^{\sqrt{2}/2} \frac{1}{1+u^2}\,du$

$\frac{1}{2}\sin^{-1}u \Big|_0^{\frac{\sqrt{2}}{2}}$

$\frac{1}{2}\left[\sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}(0)\right]$

$\frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \boxed{\frac{\pi}{8}}$

(7) $\int_0^2 \frac{1}{8+2x^2}\,dx = \int_0^2 \frac{1}{\left(\frac{8}{8}+\frac{2x^2}{8}\right)8}\,dx = \frac{1}{8}\int_0^2 \frac{1}{1+\left(\frac{x}{2}\right)^2}$

$u = \frac{x}{2}$

$du = \frac{1}{2}\,dx$

$dx = 2\,du$

$= \frac{1}{8}\int_0^1 \frac{1}{1+u^2}\,du\cdot 2 = \frac{1}{4}\int \frac{1}{1+u^2}\,du$

$\frac{1}{4}\tan^{-1}u \Big|_0^1 = \frac{1}{4}\left[\tan^{-1}1 + \tan^{-1}0\right]$

$= \frac{1}{4}\cdot\left[\frac{\pi}{4} - 0\right]$

$= \boxed{\pi/16}$

⑧ $\int_{-2}^{2} \frac{1}{4+3x^2}\,dx = \int_{-2}^{2} \frac{1}{\left(\frac{4}{4}+\frac{3x^2}{4}\right)4}\,dx$

$= \frac{1}{4}\int \frac{1}{1+\left(\frac{\sqrt{3}}{2}x\right)^2}\,dx$ $\qquad u = \frac{\sqrt{3}}{2}x$

$du = \frac{\sqrt{3}}{2}\,dx$

$= \frac{1}{4}\int \frac{1}{1+u^2}\cdot\frac{2}{\sqrt{3}}\,du$ $\qquad \frac{2}{\sqrt{3}}du = dx$

$= \frac{1}{2\sqrt{3}}\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{1+u^2}\,du = \frac{1}{2\sqrt{3}}\left[\tan^{-1}u\right]\Big|_{-\sqrt{3}}^{\sqrt{3}}$

$= \frac{1}{2\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1}(-\sqrt{3})\right]$

$= \frac{1}{2\sqrt{3}}\left[\frac{\pi}{3} + \frac{\pi}{3}\right]$

$= \frac{1}{2\sqrt{3}}\left[\frac{2\pi}{3}\right]$

$= \frac{\pi}{3\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\pi\sqrt{3}}{9}}$

(9) $\int \frac{1}{2+(x-1)^2}\,dx = \int \frac{1}{\left(\frac{2}{2}+\frac{(x-1)^2}{2}\right)2}\,dx$

$= \frac{1}{2}\int \frac{1}{1+\left(\frac{x-1}{\sqrt{2}}\right)^2}\,dx$

$u = \frac{x-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}u - \frac{1}{\sqrt{2}}$

$du = \frac{1}{\sqrt{2}}\,dx$

$dx = \sqrt{2}\,du$

$= \frac{1}{2}\int \frac{1}{1-u^2}\sqrt{2}\,du$

$= \frac{1}{2}\sqrt{2}\int \frac{1}{1-u^2}\,du$

$\frac{\sqrt{2}}{2}\tan^{-1}u + C$

$\boxed{\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C}$

(10) $\int \frac{6}{\sqrt{4-(x+1)^2}}\,dx \quad \int \frac{6}{\sqrt{\left(\frac{4}{4}-\frac{(x+1)^2}{4}\right)4}}\,dx$

$= \int \frac{6}{2\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx = 3\int \frac{1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx$

$= 3\int \frac{1}{\sqrt{1-u^2}}\,2\,du$

$u = \frac{x+1}{2}$

$du = \frac{1}{2}dx$

$dx = 2du$

$= 6\int \frac{1}{\sqrt{1-u^2}}\,du$

$= 6\sin^{-1}u + C$

$\boxed{= 6\sin^{-1}\left(\frac{x+1}{2}\right) + C}$

(9) $\displaystyle\int \frac{1}{2+(x-1)^2}\,dx = \int \frac{1}{2\left(1+\frac{(x-1)^2}{2}\right)}\,dx$

$\displaystyle = \frac{1}{2}\int \frac{1}{1+\left(\frac{x-1}{\sqrt{2}}\right)^2}\,dx$ $\qquad u = \frac{x-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}$

$du = \frac{1}{\sqrt{2}}\,dx$

$\displaystyle = \frac{1}{2}\int \frac{1}{1+u^2}\sqrt{2}\,du$ $\qquad dx = \sqrt{2}\,du$

$\displaystyle = \frac{\sqrt{2}}{2}\int \frac{1}{1+u^2}\,du$

$\displaystyle \frac{\sqrt{2}}{2}\tan^{-1}u + C$

$\displaystyle \frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$

(10) $\displaystyle\int \frac{6}{\sqrt{4-(x+1)^2}}\,dx \qquad \int \frac{6}{\sqrt{4\left(1-\frac{(x+1)^2}{4}\right)}}\,dx$

$\displaystyle = \int \frac{6}{2\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx = 3\int \frac{1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\,dx$

$u = \frac{x+1}{2}$ $\qquad \displaystyle = 3\int \frac{1}{\sqrt{1-u^2}}\,2\,du$

$du = \frac{1}{2}\,dx$ $\qquad \displaystyle = 6\int \frac{1}{\sqrt{1-u^2}}\,du$

$dx = 2\,du$ $\qquad = 6\sin^{-1}u + C$

$\displaystyle = 6\sin^{-1}\left(\frac{x+1}{2}\right) + C$

(11) $$\int_{-\pi/2}^{\pi/2} \frac{2\cos\theta}{1+\sin^2\theta}\, d\theta$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$d\theta = \frac{du}{\cos\theta}$

$$\int \frac{2\cos\theta}{1+u^2} \frac{du}{\cos\theta}$$

$\theta = \pi/2 \quad u = \sin \pi/2$

$u = 1$

$\theta = -\pi/2 \quad u = \sin -\pi/2$

$u = -1$

$$2\int_{-1}^{1} \frac{1}{1+u^2}\, du$$

$$2\tan^{-1} u \Big|_{-1}^{1}$$

$$2\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$$

$$2\left[\pi/4 - -\pi/4\right] = 2\left(\frac{\pi}{2}\right) = \pi$$

$\cot \pi/4 = 1$
$\cot \pi/6 =$

(12) $\int_{\pi/6}^{\pi/4} \frac{\csc^2\theta}{1+\cot^2\theta} \, d\theta$

$u = \cot\theta$
$du = -\csc^2\theta \, d\theta$

$d\theta = \frac{du}{-\csc^2\theta}$

$\int \frac{\csc^2\theta}{1+u^2} \frac{du}{-\csc^2\theta} = -\int_{\sqrt{3}}^{1} \frac{1}{1+u^2} \, du = \int_{1}^{\sqrt{3}} \frac{1}{1+u^2} \, du$

$\tan^{-1} u \Big|_{1}^{\sqrt{3}}$

$\tan^{-1}(1) - \tan^{-1}(\sqrt{3})$

$\pi/4 - \pi/6 = \frac{3\pi}{12} - \frac{2\pi}{12} = \boxed{\pi/12}$

(13) $\displaystyle\int_0^{\ln\sqrt{3}} \frac{e^x}{1+(e^x)^2}\,dx$

$u = e^x$

$du = e^x\,dx$

$dx = \dfrac{du}{e^x}$

$$\int_1^{\sqrt{3}} \frac{\cancel{e^x}}{1+u^2}\,\frac{du}{\cancel{e^x}} = \tan^{-1}u\,\Big|_1^{\sqrt{3}}$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{4\pi}{12} - \frac{3\pi}{12}$$

$$= \boxed{\frac{\pi}{12}}$$

(14) $$\int_1^{e^{\pi/4}} \frac{4}{x(1+(\ln x)^2)}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$dx = x\,du$

$$4\int_0^{\pi/4} \frac{1}{\cancel{x}(1+u^2)} \cdot \cancel{x}\,du$$

$$4\tan^{-1}u \Big|_0^{\pi/4}$$

$$4\left[\tan^{-1}\frac{\pi}{4} - \tan^{-1}0\right] = \boxed{4\tan^{-1}\left(\frac{\pi}{4}\right)}$$

(15) $$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$dx = \frac{du}{\sec^2 x}$

$$\int \frac{\cancel{\sec^2 x}}{\sqrt{1-u^2}} \frac{du}{\cancel{\sec^2 x}}$$

$$\boxed{\sin^{-1}(\tan x) + c}$$

(16) $\int_0^1 x e^{x^2} dx$

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \cancel{x} e^u \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2} e^u \Big|_0^1 = \frac{1}{2}[e - e^0] = \frac{1}{2}[e-1]$

(17) $\int_0^{\ln 4} -e^{-2x} dx$

$u = -2x$

$du = -2\,dx$

$dx = \frac{du}{-2}$

$-2\ln 4$

$\ln 4^{-2}$

$\int -e^u \frac{du}{-2}$

$= \frac{1}{2}\int_0^{\ln 4^{-2}} e^u\,du \quad \frac{1}{2} e^u \Big|_0^{\ln 4^{-2}} = 1/2\left[e^{\ln 4^{-2}} - e^0\right]$

$= \frac{1}{2}\left[\frac{1}{16} - 1\right]$

$= \frac{1}{2}\left[\frac{-15}{16}\right]$

$= \frac{-15}{32}$

(18) $\int_0^{\pi/2} \frac{\cos x}{\sin x + 2}\, dx$

$u = \sin x + 2$

$du = \cos x\, dx$

$dx = \frac{du}{\cos x}$

$\int_2^3 \frac{\cos x}{u} \cdot \frac{du}{\cos x}\, dx = \ln u \Big|_2^3 = \ln 3 - \ln 2$

$= \ln \frac{3}{2}$

(19) $\int_0^2 \frac{x^2}{x^3+2}\, dx$

$u = x^3 + 2$

$du = 3x^2\, dx$

$dx = \frac{du}{3x^2}$

$\int \frac{x^2}{u} \frac{du}{3x^2}$

$\frac{1}{3} \int_2^{10} \frac{1}{u}\, du = \frac{1}{3} \ln u \Big|_2^{10} = \frac{1}{3}(\ln 10 - \ln 2)$

$= \frac{1}{3} \ln 5$

(20) $\int \frac{1+\ln x}{x}\,dx = \int \frac{u}{x}\cdot x\,du = \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{1}{2}(1+\ln x)^2 + C$

$u = 1+\ln x$

$du = \frac{1}{x}dx$

$dx = x\,du$

(21) $\int \frac{e^x}{e^x+4}\,dx = \int \frac{e^x}{u}\cdot\frac{1}{e^x}\,du = \int \frac{1}{u}\,du = \ln|u| + C$

$= \ln|e^x+4| + C$

$u = e^x+4$

$du = e^x\,dx$

$dx = \frac{1}{e^x}\,du$

(22) ~~$\int_1^2 \frac{x^2+4}{x^3}\,dx = \int \frac{x^2+4}{u}\,\frac{1}{3x^2}\,du$~~

~~$u = x^3$~~

~~$du = 3x^2\,dx$~~

$dx = \frac{1}{3x^2}\,du$

$4x^{-3}$ $\quad \frac{4x^{-2}}{-2}$ $\quad -2x^{-2}$

$\int_1^2 \frac{x^2+4}{x^3}\,dx = \int_1^2 \left(\frac{1}{x}+\frac{4}{x^3}\right)dx$

$=$ ~~$\ln|x| + \frac{2}{x^2}\Big|_1^2$~~

$= \ln|x| - \frac{2}{x^2}\Big|_1^2$

$= \ln 2 - \frac{2}{2^2} - \left(\ln 1 - \frac{2}{1^2}\right)$

$\ln 2 - \frac{1}{2} - 0 + 2$

$\ln 2 + 3/2$

(23) $\int \frac{e^x+2}{e^x}\,dx = \int 1 + 2e^{-x}\,dx$

$u = -x$

$du = -dx$

$= x + -\int 2e^u\,du$

$= x - 2e^u + C$

$= x - 2e^{-x} + C$

(24) $\int \frac{x+3}{x-1}\,dx$ $\quad \int \frac{x+3}{u}\,du = \int \frac{u+1+3}{u}\,du$

$u = x-1$

$du = dx$

$x = u+1$

$= \int \frac{u+4}{u}\,du$

$= \int 1 + \frac{4}{u}\,du$

$= u + 4\ln|u|$

$= (x-1) + 4\ln|x-1| + C$

or

$x + 4\ln|x-1| + C$

① $\int \frac{1}{\sqrt{9-x^2}}dx = \int \frac{1}{\sqrt{9(1-\frac{x^2}{9})}}\,dx = \int \frac{1}{3\sqrt{1-(\frac{x}{3})^2}}\,dx =$

$u = \frac{x}{3}$
$du = \frac{1}{3}dx$
$dx = 3\,du$

$\int \frac{1}{3\sqrt{1-u^2}} \cdot 3\,du = \int \frac{1}{\sqrt{1-u^2}}\,du$

$= \sin^{-1} u + c$

$= \boxed{\sin^{-1}(\frac{x}{3}) + c}$

② $\int \frac{1}{\sqrt{1-(2x)^2}}\,dx$

$u = 2x$
$du = 2dx$
$dx = \frac{1}{2}du$

$= \int \frac{1}{\sqrt{1-u^2}}\,\frac{1}{2}du = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1}(u) + c$

$= \boxed{\frac{1}{2}\sin^{-1}(2x) + c}$

③ $\int \frac{1}{17+x^2}\,dx = \int \frac{1}{17(1+\frac{x^2}{17})}\,dx = \frac{1}{17}\int \frac{1}{1+(\frac{x}{\sqrt{17}})^2}\,dx$

$u = \frac{x}{\sqrt{17}}$
$du = \frac{1}{\sqrt{17}}dx$
$dx = \sqrt{17}\,du$

$= \frac{1}{17}\int \frac{1}{1+u^2}\sqrt{17}\,du$

$= \frac{\sqrt{17}}{17}\int \frac{1}{1+u^2}\,du = \frac{\sqrt{17}}{17}\tan^{-1} u + c$

$= \boxed{\frac{\sqrt{17}}{17}\tan^{-1}\left(\frac{x}{\sqrt{17}}\right) + c}$

(4) $$\int \frac{1}{9+3x^2}\,dx = \int \frac{1}{9\left(1+\frac{x^2}{3}\right)}\,dx = \int \frac{1}{9\left(1+\left(\frac{x}{\sqrt{3}}\right)^2\right)}\,dx$$

$u = \frac{x}{\sqrt{3}}$

$du = \frac{1}{\sqrt{3}}dx$

$dx = \sqrt{3}\,du$

$$= \int \frac{1}{9(1+u^2)}\sqrt{3}\,du = \frac{\sqrt{3}}{9}\int \frac{1}{1+u^2}\,du =$$

$$= \frac{\sqrt{3}}{9}\tan^{-1}u + c = \boxed{\frac{\sqrt{3}}{9}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c}$$

(5) $$\int_0^1 \frac{4}{\sqrt{4-x^2}}\,dx = \int_0^1 \frac{4}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}\,dx = \int_0^1 \frac{4}{2\left(1+\left(\frac{x}{2}\right)^2\right)}\,dx$$

$$= \int \frac{4}{2(1+u^2)}\cdot 2\,du = 4\int_0^{1/2} \frac{1}{1+u^2}\,du$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$dx = 2\,du$

$x=0;\ u=0$

$x=1;\ u=1/2$

$$= 4\sin^{-1}u\Big|_0^{1/2} = 4\left[\sin^{-1}\tfrac{1}{2} - \sin^{-1}0\right]$$

$$= 4\left[\frac{\pi}{6} - 0\right]$$

$$= \frac{4\pi}{6} = \boxed{\frac{2\pi}{3}}$$

(6) $$\int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9-4x^2}}\,dx = \int \frac{1}{\sqrt{9\left(1-\frac{4x^2}{9}\right)}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-\left(\frac{2x}{3}\right)^2}}\,dx$$

$u = \frac{2x}{3}$

$du = 2/3\,dx$

$dx = 3/2\,du$

$x=0: u=0$

$x = \frac{3\sqrt{2}}{4}: u = \frac{2\cdot\frac{3\sqrt{2}}{4}}{3}$

$\frac{\sqrt{2}}{2}$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\cdot\frac{3}{2}\,du = \frac{1}{2}\int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\sin^{-1}u\Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{2}\left[\sin^{-1}\frac{2}{\sqrt{2}} - \sin^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - 0\right]$$

$$= \boxed{\frac{\pi}{8}}$$

(7) $$\int_0^2 \frac{1}{8+2x^2}\,dx = \int_0^2 \frac{1}{8\left(1+\frac{2x^2}{8}\right)}\,dx = \frac{1}{8}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx$$

$$= \frac{1}{8}\int \frac{1}{1+u^2}\cdot 2\,du$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$dx = 2\,du$

$$= \frac{1}{4}\int_0^1 \frac{1}{1+u^2}\,du = \frac{1}{4}\tan^{-1}u\Big|_0^1$$

$x=0: u = \frac{0}{2} = 0$

$x=2: u = \frac{2}{2} = 1$

$$= \frac{1}{4}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{4} - 0\right]$$

$$= \boxed{\pi/16}$$