

$$(1) \int \frac{1}{\sqrt{9-x^2}} = \int \frac{1}{\sqrt{\left(\frac{9-x^2}{9}\right)9}} dx = \int \frac{1}{3\sqrt{1-\left(\frac{x}{3}\right)^2}} dx$$

$$\int \frac{1}{3\sqrt{1-u^2}} \cdot 3 du = \int \frac{1}{\sqrt{1-u^2}} du$$

$$u = \frac{x}{3}$$
$$du = \frac{1}{3} dx$$
$$dx = 3 du$$

$$= \sin^{-1} u + C$$

$$= \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$(2) \int \frac{1}{\sqrt{1-(2x)^2}} dx \quad - \quad u = 2x$$
$$du = 2 dx$$
$$dx = \frac{du}{2}$$

$$= \int \frac{1}{\sqrt{1-u^2}} \frac{du}{2} = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \sin^{-1}(u) + C$$

$$= \frac{1}{2} \sin^{-1}(2x) + C$$

$$\int \frac{1}{\sqrt{2x^2-1}} dx = \int \frac{1}{\sqrt{2(x^2-\frac{1}{2})}} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2-\frac{1}{2}}} dx \quad (1)$$

$\frac{1}{\sqrt{2}} = u$
 $2x = 2u$
 $2dx = 2du$

$$\int \frac{1}{\sqrt{2x^2-1}} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2-\frac{1}{2}}} dx$$

$$= \frac{1}{\sqrt{2}} \ln |x + \sqrt{x^2-\frac{1}{2}}| + C$$

$$= \frac{1}{\sqrt{2}} \ln |x + \sqrt{x^2-\frac{1}{2}}| + C$$

$$\int \frac{1}{\sqrt{1-5x^2}} dx \quad (2)$$

$u = 5x$
 $5dx = du$
 $dx = \frac{du}{5}$

$$\int \frac{1}{\sqrt{1-5x^2}} dx = \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{5} du = \frac{1}{5} \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \frac{1}{5} \arcsin(u) + C = \frac{1}{5} \arcsin(5x) + C$$

$$\textcircled{3} \int \frac{1}{17+x^2} dx = \int \frac{1}{\left(\frac{17}{17} + \frac{x^2}{17}\right)} \frac{1}{17} dx \quad \textcircled{1}$$

$$= \frac{1}{17} \int \frac{1}{1 + \left(\frac{x}{\sqrt{17}}\right)^2} dx$$

$$u = \frac{x}{\sqrt{17}}$$

$$du = \frac{1}{\sqrt{17}} dx$$

$$dx = \sqrt{17} du$$

$$= \frac{1}{17} \int \frac{1}{1+u^2} \cdot \sqrt{17} du$$

$$= \frac{\sqrt{17}}{17} \int \frac{1}{1+u^2} du$$

$$= \frac{\sqrt{17}}{17} \tan^{-1} u + C$$

$$= \frac{\sqrt{17}}{17} \tan^{-1} \left(\frac{x}{\sqrt{17}} \right) + C \quad \textcircled{2}$$

$$\textcircled{4} \int \frac{1}{9+3x^2} dx = \int \frac{1}{3(3+x^2)} dx$$

$$= \frac{1}{3} \int \frac{1}{\left(\frac{3}{3} + \frac{x^2}{3}\right) 3} = \frac{1}{9} \int \frac{1}{1 + \frac{1}{3}x^2} = \frac{1}{9} \int \frac{1}{1 + \left(\frac{x}{\sqrt{3}}\right)^2}$$

$$u = \frac{x}{\sqrt{3}}$$

$$du = \frac{1}{\sqrt{3}} dx$$

$$dx = \sqrt{3} du$$

$$= \frac{1}{9} \int \frac{1}{1+u^2} du \cdot \sqrt{3}$$

$$= \frac{\sqrt{3}}{9} \tan^{-1} u + C$$

$$= \frac{\sqrt{3}}{9} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + C$$

$$\textcircled{5} \int_0^1 \frac{4}{\sqrt{4-x^2}} dx = \int_0^1 \frac{4}{\sqrt{\left(\frac{4}{4} + \frac{x^2}{4}\right) 4}} dx$$

$$= \int_0^1 \frac{4}{2\sqrt{1+\left(\frac{x}{2}\right)^2}} dx$$

$$u = \frac{x}{2}$$

$$du = \frac{1}{2} dx$$

$$dx = du \cdot 2$$

$$= \int \frac{2}{\sqrt{1+u^2}} du \cdot 2$$

$$= 4 \int_0^{1/2} \frac{1}{\sqrt{1+u^2}}$$

$$4 \sin^{-1} u \Big|_0^{1/2}$$

$$= 4 \left[\sin^{-1} \frac{1}{2} - \sin^{-1} 0 \right]$$

$$= 4 \left[\frac{\pi}{6} - 0 \right]$$

$$= \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\textcircled{6} \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9-4x^2}} dx = \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\left(\frac{9}{4} + \frac{4x^2}{9}\right)^{\frac{1}{2}}} dx = \frac{1}{3} \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{1 + \left(\frac{2}{3}x\right)^2} dx$$

$$u = \frac{2}{3}x \quad \times \frac{3}{2} \cdot \frac{1}{3} \int \frac{1}{1+u^2} du = \frac{1}{2} \int \frac{1}{1+u^2} du$$

$$du = \frac{2}{3} dx$$

$$dx = \frac{3}{2} du$$

$$\frac{1}{2} \sin^{-1} u \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$\frac{1}{2} \left[\sin^{-1} \frac{\sqrt{2}}{2} - \sin^{-1}(0) \right]$$

$$\frac{1}{2} \left[\frac{\pi}{4} - 0 \right] = \textcircled{\frac{\pi}{8}}$$

$$\textcircled{7} \int_0^2 \frac{1}{8+2x^2} dx = \int_0^2 \frac{1}{\left(\frac{8}{2} + \frac{2x^2}{2}\right) 2} dx = \frac{1}{8} \int_0^2 \frac{1}{1 + \left(\frac{x}{2}\right)^2} dx$$

$$u = \frac{x}{2} \quad = \frac{1}{8} \int_0^1 \frac{1}{1+u^2} du \cdot 2 = \frac{1}{4} \int_0^1 \frac{1}{1+u^2} du$$

$$du = \frac{1}{2} dx$$

$$dx = 2 du$$

$$\frac{1}{4} \tan^{-1} u \Big|_0^1 = \frac{1}{4} \left[\tan^{-1} 1 + \tan^{-1} 0 \right]$$

$$= \frac{1}{4} \left[\frac{\pi}{4} - 0 \right]$$

$$= \textcircled{\frac{\pi}{16}}$$

$$\textcircled{8} \int_{-2}^2 \frac{1}{4+3x^2} dx = \int_{-2}^2 \frac{1}{\left(\frac{4}{4} + \frac{3x^2}{4}\right)4} dx$$

$$= \frac{1}{4} \int \frac{1}{1 + \left(\frac{\sqrt{3}}{2}x\right)^2} dx \quad u = \frac{\sqrt{3}}{2}x$$

$$du = \frac{\sqrt{3}}{2} dx$$

$$= \frac{1}{4} \int \frac{1}{1+u^2} \cdot \frac{2}{\sqrt{3}} du = \frac{2}{4\sqrt{3}} \int \frac{1}{1+u^2} du = \frac{1}{2\sqrt{3}} \int \frac{1}{1+u^2} du$$

$$= \frac{1}{2\sqrt{3}} \int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{1+u^2} du = \frac{1}{2\sqrt{3}} \left[\tan^{-1} u \right]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}} \left[\tan^{-1} \sqrt{3} - \tan^{-1}(-\sqrt{3}) \right]$$

$$= \frac{1}{2\sqrt{3}} \left[\frac{\pi}{3} + \frac{\pi}{3} \right]$$

$$= \frac{1}{2\sqrt{3}} \left[\frac{2\pi}{3} \right]$$

$$= \frac{\pi}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\pi\sqrt{3}}{3}$$

$$\textcircled{9} \int \frac{1}{2+(x-1)^2} dx = \int \frac{1}{\left(\frac{2}{2} + \frac{(x-1)^2}{2}\right) 2} dx$$

$$= \frac{1}{2} \int \frac{1}{1+\left(\frac{x-1}{\sqrt{2}}\right)^2} dx$$

$$u = \frac{x-1}{\sqrt{2}} = \frac{1}{\sqrt{2}} u - \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2} \int \frac{1}{1-u^2} \sqrt{2} du$$

$$du = \frac{1}{\sqrt{2}} dx$$

$$dx = \sqrt{2} du$$

$$= \frac{1}{2} \sqrt{2} \int \frac{1}{1-u^2} du$$

$$\frac{\sqrt{2}}{2} \tan^{-1} u + C$$

$$\frac{\sqrt{2}}{2} \tan^{-1} \left(\frac{x-1}{\sqrt{2}} \right) + C$$

$$\textcircled{10} \int \frac{6}{\sqrt{4-(x+1)^2}} dx = \int \frac{6}{\sqrt{\left(\frac{4}{4} - \frac{(x+1)^2}{4}\right) 4}} dx$$

$$= \int \frac{6}{2 \sqrt{1-\left(\frac{x+1}{2}\right)^2}} dx = 3 \int \frac{1}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}} dx$$

$$= 3 \int \frac{1}{\sqrt{1-u^2}} 2 du$$

$$u = \frac{x+1}{2}$$

$$= 6 \int \frac{1}{\sqrt{1-u^2}} du$$

$$du = \frac{1}{2} dx$$

$$= 6 \sin^{-1} u + C$$

$$dx = 2 du$$

$$= 6 \sin^{-1} \left(\frac{x+1}{2} \right) + C$$

$$x \cdot \left[\frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \right] = x \cdot \frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \quad (1)$$

$$\frac{1}{s^2 - x^2} = \frac{1}{s^2 - \left(\frac{x}{5}\right)^2} = 1$$

$$x \cdot \left[\frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \right] = \frac{1}{s^2}$$

$$x \cdot \frac{1}{s^2} = \frac{1}{s^2} \Rightarrow x = 1$$

$$\frac{1}{s^2} = \frac{1}{s^2} \Rightarrow x = 1$$

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$$x \cdot \left[\frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \right] = x \cdot \frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \quad (2)$$

$$x \cdot \left[\frac{1}{s^2 + \left(\frac{x}{5}\right)^2} \right] = x \cdot \frac{1}{s^2 + \left(\frac{x}{5}\right)^2} =$$

$$\frac{1}{s^2} = \frac{1}{s^2} \Rightarrow x = 1$$

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$$(11) \int_{-\pi/2}^{\pi/2} \frac{2 \cos \theta}{1 + \sin^2 \theta} d\theta$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$d\theta = \frac{du}{\cos \theta}$$

(51)

$$\int \frac{2 \cancel{\cos \theta}}{1 + u^2} \frac{du}{\cancel{\cos \theta}}$$

$$\theta = \pi/2 \quad u = \sin \pi/2$$

$$u = 1$$

$$\theta = -\pi/2 \quad u = \sin -\pi/2$$

$$u = -1$$

$$2 \int_{-1}^1 \frac{1}{1+u^2} du$$

$$2 \tan^{-1} u \Big|_{-1}^1$$

$$2 [\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$2 \left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) \right] = 2 \left(\frac{\pi}{2} \right) = \pi$$

$$\cot \pi/4 = 1$$

$$\cot \pi/6 = \sqrt{3}$$

$$\pi/4$$

$$\textcircled{12} \int_{\pi/6}^{\pi/4} \frac{\csc^2 \theta}{1 + \cot^2 \theta} d\theta$$

$$u = \cot \theta$$

$$du = -\csc^2 \theta d\theta$$

$$d\theta = \frac{du}{-\csc^2 \theta}$$

$$\int \frac{\csc^2 \theta}{1 + u^2} \frac{du}{-\csc^2 \theta} = - \int \frac{1}{1 + u^2} du = - \int \frac{1}{1 + u^2} du$$

$$+ \tan^{-1} u \Big|_1^{\sqrt{3}}$$

$$\tan^{-1}(1) - \tan^{-1}(\sqrt{3})$$

$$\pi/4 - \pi/6 = \frac{3\pi}{12} - \frac{2\pi}{12} = \textcircled{\pi/12}$$

$$\textcircled{13} \int_0^{\ln \sqrt{3}} \frac{e^x}{1+(e^x)^2} dx$$

$$u = e^x$$

$$du = e^x dx$$

$$dx = \frac{du}{e^x}$$

$$\int_1^{\sqrt{3}} \frac{e^x}{1+u^2} \frac{du}{e^x} = \tan^{-1} u \Big|_1^{\sqrt{3}}$$

$$= \tan^{-1} \sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{4\pi}{12} - \frac{3\pi}{12}$$

$$= \textcircled{\frac{\pi}{12}}$$

$$\begin{aligned}
 x^2 &= a \\
 x b^x &= ab \\
 ab &= x b \\
 \frac{ab}{b} &= \frac{x b}{b} \\
 a &= x
 \end{aligned}$$

$$\left. \begin{aligned}
 x^2 &= a \\
 x b^x &= ab \\
 ab &= x b
 \end{aligned} \right\} \textcircled{15}$$

$$\frac{1}{x} = \frac{1}{x}$$

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$$\frac{1}{x} = \frac{1}{x}$$

$$\textcircled{15}$$

$$(14) \int_1^{e^{\pi/4}} \frac{4}{x(1+(\ln x)^2)} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dx = x du$$

$$4 \int_0^{\pi/4} \frac{1}{x(1+u^2)} \cdot x du$$

$$4 \tan^{-1} u \Big|_0^{\pi/4}$$

$$4 \left(\tan^{-1} \frac{\pi}{4} - \tan^{-1} 0 \right) = 4 \tan^{-1} \left(\frac{\pi}{4} \right)$$

~~4 \tan^{-1} \frac{\pi}{4} - 4 \tan^{-1} 0~~

~~4 \tan^{-1} \frac{\pi}{4}~~

$$(15) \int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$$

$$u = \tan x$$

$$du = \sec^2 x dx$$

$$dx = \frac{du}{\sec^2 x}$$

$$\int \frac{\cancel{\sec^2 x}}{\sqrt{1-u^2}} \frac{du}{\cancel{\sec^2 x}}$$

~~4 \tan^{-1} \frac{\pi}{4} - 4 \tan^{-1} 0~~

$$\sin^{-1}(\tan x) + c$$

$$x \cdot 1 = x$$

$$x \cdot \frac{1}{x} = 1$$

$$x \cdot x = x^2$$

$$x^b \left[\begin{array}{c} 1 \\ (x^{(n+1)+1})^x \end{array} \right] \quad (11)$$

$$x^b \left[\begin{array}{c} 1 \\ (x^{(n+1)+1})^x \end{array} \right] \quad (12)$$

$$(11) \cdot (12) = \left(\begin{array}{c} 1 \\ 0 \end{array} \right) \cdot \left(\begin{array}{c} 1 \\ 1 \end{array} \right) = \left(\begin{array}{c} 1 \\ 1 \end{array} \right)$$

~~Handwritten scribbles~~

$$N^{-1} \cdot \text{not } H$$

$$x \cdot 1 = x$$

$$x \cdot \frac{1}{x} = 1$$

$$x \cdot x = x^2$$

$$x^b \left[\begin{array}{c} 1 \\ (1-x)^x \end{array} \right] \quad (13)$$

$$x^b \left[\begin{array}{c} 1 \\ (1-x)^x \end{array} \right]$$

$$x^b = x^b$$

$$\text{of } (x+1)^{-1} \cdot x^b$$

(16) $\int_0^1 x e^{x^2} dx$ (8)

$u = x^2$
 $du = 2x dx$
 $dx = \frac{du}{2x}$

$\int x e^u \frac{du}{2x}$

$\frac{1}{2} \int_0^1 e^u du = \frac{1}{2} e^u \Big|_0^1 = \frac{1}{2} [e - e^0] = \frac{1}{2} [e - 1]$

(17) $\int_0^{\ln 4} -e^{-2x} dx$ (9)

$u = -2x$
 $du = -2 dx$
 $dx = \frac{du}{-2}$

$\int -e^u \frac{du}{-2}$

$= \frac{1}{2} \int_0^{\ln 4} e^u du = \frac{1}{2} e^u \Big|_0^{\ln 4} = \frac{1}{2} [e^{\ln 4} - e^0]$

$= \frac{1}{2} \left[\frac{1}{16} - 1 \right]$

$= \frac{1}{2} \left[\frac{-15}{16} \right]$

$= \frac{-15}{32}$

(18) $\int_0^{\pi/2} \frac{\cos x}{\sin x + 2} dx$

$u = \sin x + 2$
 $du = \cos x dx$
 $dx = \frac{du}{\cos x}$

$\int_2^3 \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}} = \ln u \Big|_2^3 = \ln 3 - \ln 2$
 $= \ln \frac{3}{2}$

(19) $\int_2^{10} \frac{x^2}{x^3 + 2} dx$

$u = x^3 + 2$
 $du = 3x^2 dx$
 $dx = \frac{du}{3x^2}$

$\int_2^{10} \frac{x^2}{u} \cdot \frac{du}{3x^2} = \frac{1}{3} \int_2^{10} \frac{1}{u} du = \frac{1}{3} \ln u \Big|_2^{10} = \frac{1}{3} (\ln 10 - \ln 2)$
 $= \frac{1}{3} \ln 5$

$$\textcircled{20} \int \frac{1+\ln x}{x} dx = \int \frac{u}{x} \cdot x du = \int u du = \frac{u^2}{2} + C$$

$$u = 1 + \ln x$$

$$du = \frac{1}{x} dx$$

$$dx = x du$$

$$= \frac{1}{2}(1+\ln x)^2 + C$$

$$\textcircled{21} \int \frac{e^x}{e^x+4} dx = \int \frac{e^x}{u} \cdot \frac{1}{e^x} du = \int \frac{1}{u} du = \ln|u| + C$$

$$u = e^x + 4$$

$$du = e^x dx$$

$$dx = \frac{1}{e^x} du$$

$$= \ln|e^x+4| + C$$

$$\textcircled{22} \int_1^2 \frac{x^2+4}{x^3} dx = \int \frac{x^2+4}{u} \cdot \frac{1}{3x^2} dx$$

$$\int_1^2 \frac{x^2+4}{x^3} dx = \int_1^2 \left(\frac{1}{x} + \frac{4}{x^3} \right) dx$$

~~$$= \ln|x| - \frac{2}{x^2} \Big|_1^2$$~~

$$= \ln|x| - \frac{2}{x^2} \Big|_1^2$$

$$= \ln 2 - \frac{2}{2^2} - \left(\ln 1 - \frac{2}{1^2} \right)$$

$$\ln 2 - \frac{1}{2} - 0 + 2$$

$$\ln 2 + \frac{3}{2}$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dx = \frac{1}{3x^2} du$$

$$4x^{-3}$$

$$\frac{4x^{-2}}{-2}$$

$$-2x^{-2}$$

$$(23) \int \frac{e^x + 2}{e^x} dx = \int 1 + 2e^{-x} dx$$

$$= x + \int 2e^u du$$

$$= x - 2e^u + C$$

$$= \boxed{x - 2e^{-x} + C}$$

$$u = -x$$

$$du = -dx$$

$$(24) \int \frac{x+3}{x-1} dx \quad \int \frac{x+3}{u} du = \int \frac{u+1+3}{u} du$$

$$u = x-1$$

$$du = dx$$

$$x = u+1$$

$$= \int \frac{u+4}{u} du$$

$$= \int 1 + \frac{4}{u} du$$

~~just~~

$$= u + 4 \ln |u|$$

$$= \boxed{(x-1) + 4 \ln |x-1| + C}$$

or

$$\underline{\underline{x + 4 \ln |x-1| + C}}$$

$$\textcircled{1} \int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{9(1-\frac{x^2}{9})}} dx = \int \frac{1}{3\sqrt{1-(\frac{x}{3})^2}} dx =$$

$$u = \frac{x}{3}$$

$$du = \frac{1}{3} dx$$

$$dx = 3 du$$

$$\int \frac{1}{3\sqrt{1-u^2}} \cdot 3 du = \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1} u + C$$

$$= \sin^{-1}\left(\frac{x}{3}\right) + C$$

$$\textcircled{2} \int \frac{1}{\sqrt{1-(2x)^2}} dx$$

$$u = 2x$$

$$du = 2 dx$$

$$dx = \frac{1}{2} du$$

$$= \int \frac{1}{\sqrt{1-u^2}} \frac{1}{2} du = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \sin^{-1}(u) + C$$

$$= \frac{1}{2} \sin^{-1}(2x) + C$$

$$\textcircled{3} \int \frac{1}{17+x^2} dx = \int \frac{1}{17(1+\frac{x^2}{17})} dx = \frac{1}{17} \int \frac{1}{1+(\frac{x}{\sqrt{17}})^2} dx$$

$$= \frac{1}{17} \int \frac{1}{1+u^2} \sqrt{17} du$$

$$u = \frac{x}{\sqrt{17}}$$

$$du = \frac{1}{\sqrt{17}} dx$$

$$dx = \sqrt{17} du$$

$$= \frac{\sqrt{17}}{17} \int \frac{1}{1+u^2} du = \frac{\sqrt{17}}{17} \tan^{-1} u + C$$

$$= \frac{\sqrt{17}}{17} \tan^{-1}\left(\frac{x}{\sqrt{17}}\right) + C$$

$$\begin{aligned} \sqrt{a} &= a^{\frac{1}{2}} \\ \sqrt{a^2} &= a^{\frac{2}{2}} = a \\ \sqrt{a^4} &= a^{\frac{4}{2}} = a^2 \end{aligned}$$

$$= ab \sqrt{\frac{1}{(a^2-1)^2}} = ab \sqrt{\frac{1}{(a-1)^2(a+1)^2}} = ab \frac{1}{(a-1)(a+1)} \quad (1)$$

$$ab \sqrt{\frac{1}{(a-1)^2}} = ab \sqrt{\frac{1}{(a-1)^2}} = \frac{ab}{a-1}$$

$$\frac{ab}{(a-1)^2} = \frac{ab}{(a-1)^2}$$

$$\begin{aligned} ab &= a \\ ab &= ab \\ ab &= ab \end{aligned}$$

$$ab \sqrt{\frac{1}{(a^2-1)^2}} \quad (2)$$

$$\frac{ab}{(a-1)^2} = ab \sqrt{\frac{1}{(a-1)^2}} = \frac{ab}{a-1}$$

$$\frac{ab}{(a-1)^2} = \frac{ab}{(a-1)^2}$$

$$\frac{ab}{(a-1)^2} = ab \sqrt{\frac{1}{(a-1)^2}} = \frac{ab}{a-1} \quad (3)$$

$$\begin{aligned} \sqrt{a} &= a^{\frac{1}{2}} \\ \sqrt{a^2} &= a \\ \sqrt{a^4} &= a^2 \end{aligned}$$

$$ab \sqrt{\frac{1}{(a-1)^2}} = \frac{ab}{a-1}$$

$$\frac{ab}{(a-1)^2} = ab \sqrt{\frac{1}{(a-1)^2}} = \frac{ab}{a-1}$$

$$\frac{ab}{(a-1)^2} = \frac{ab}{(a-1)^2}$$

$$\begin{aligned}
 \textcircled{4} \int \frac{1}{9+3x^2} dx &= \int \frac{1}{9(1+\frac{x^2}{3})} dx = \int \frac{1}{9(1+(\frac{x}{\sqrt{3}})^2)} dx & u = \frac{x}{\sqrt{3}} \\
 & & du = \frac{1}{\sqrt{3}} dx \\
 & & dx = \sqrt{3} du \\
 &= \int \frac{1}{9(1+u^2)} \sqrt{3} du = \frac{\sqrt{3}}{9} \int \frac{1}{1+u^2} du = \\
 &= \frac{\sqrt{3}}{9} \tan^{-1} u + C = \frac{\sqrt{3}}{9} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + C
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \int_0^1 \frac{4}{\sqrt{4-x^2}} dx &= \int_0^1 \frac{4}{\sqrt{4(1-\frac{x^2}{4})}} dx = \int_0^1 \frac{4}{2(1+(\frac{x}{2})^2)} dx \\
 &= \int_0^1 \frac{4}{2(1+u^2)} \cdot 2 du = 4 \int_0^{1/2} \frac{1}{1+u^2} du & u = \frac{x}{2} \\
 & & du = \frac{1}{2} dx \\
 & & dx = 2 du \\
 & & x=0; u=0 \\
 & & x=1; u=1/2 \\
 &= 4 \sin^{-1} u \Big|_0^{1/2} = 4 \left[\sin^{-1} \frac{1}{2} - \sin^{-1} 0 \right] \\
 &= 4 \left[\frac{\pi}{6} - 0 \right] \\
 &= \frac{4\pi}{6} = \frac{2\pi}{3}
 \end{aligned}$$

$\frac{x}{x^2-1}$
 $\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$
 $\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1} \quad (1)$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1} \quad (2)$$

$\frac{x}{x^2-1}$
 $\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$
 $\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\frac{x}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}$$

$$\textcircled{6} \int_0^{\frac{3\sqrt{2}}{4}} \frac{1}{\sqrt{9-4x^2}} dx = \int \frac{1}{\sqrt{9(1-\frac{4x^2}{9})}} dx = \frac{1}{3} \int \frac{1}{\sqrt{1-(\frac{2x}{3})^2}} dx$$

$$\frac{1}{3} \int \frac{1}{\sqrt{1-u^2}} \cdot \frac{3}{2} du = \frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \sin^{-1} u \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{2} \left[\sin^{-1} \frac{\sqrt{2}}{2} - \sin^{-1}(0) \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{4} - 0 \right]$$

$$= \frac{\pi}{8}$$

$$u = \frac{2x}{3}$$

$$du = \frac{2}{3} dx$$

$$dx = \frac{3}{2} du$$

$$x=0: u=0$$

$$x = \frac{3\sqrt{2}}{4}: u = \frac{2 \cdot \frac{3\sqrt{2}}{4}}{3} = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2}$$

$$\textcircled{7} \int_0^2 \frac{1}{8+2x^2} dx = \int_0^2 \frac{1}{8(1+\frac{2x^2}{8})} dx = \frac{1}{8} \int \frac{1}{1+(\frac{x}{2})^2} dx$$

$$= \frac{1}{8} \int \frac{1}{1+u^2} \cdot 2 du$$

$$= \frac{1}{4} \int_0^1 \frac{1}{1+u^2} du = \frac{1}{4} \tan^{-1} u \Big|_0^1$$

$$= \frac{1}{4} \left[\tan^{-1}(1) - \tan^{-1}(0) \right]$$

$$= \frac{1}{4} \left[\frac{\pi}{4} - 0 \right]$$

$$= \frac{\pi}{16}$$

$$u = \frac{x}{2}$$

$$du = \frac{1}{2} dx$$

$$dx = 2 du$$

$$x=0: u = \frac{0}{2} = 0$$

$$x=2: u = \frac{2}{2} = 1$$

$$x^2 \frac{d}{dx} \left(\frac{1}{x^2} \right) = x^2 \left(\frac{-2}{x^3} \right) = -\frac{2}{x} \quad (4)$$

$$\frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3} = -2x^{-3}$$

$$\left[\frac{1}{x^2} = x^{-2} \right]$$

$$\left[-2x^{-3} \right]$$

$$\left(\frac{1}{x^3} \right)$$

$$x^2 \frac{d}{dx} \left(\frac{1}{x} \right) = x^2 \left(\frac{-1}{x^2} \right) = -1 \quad (5)$$

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$\left[\frac{1}{x} = x^{-1} \right]$$

$$\frac{d}{dx} x^{-1} = -1x^{-2} = -\frac{1}{x^2}$$

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$\left(\frac{1}{x^2} \right)$$