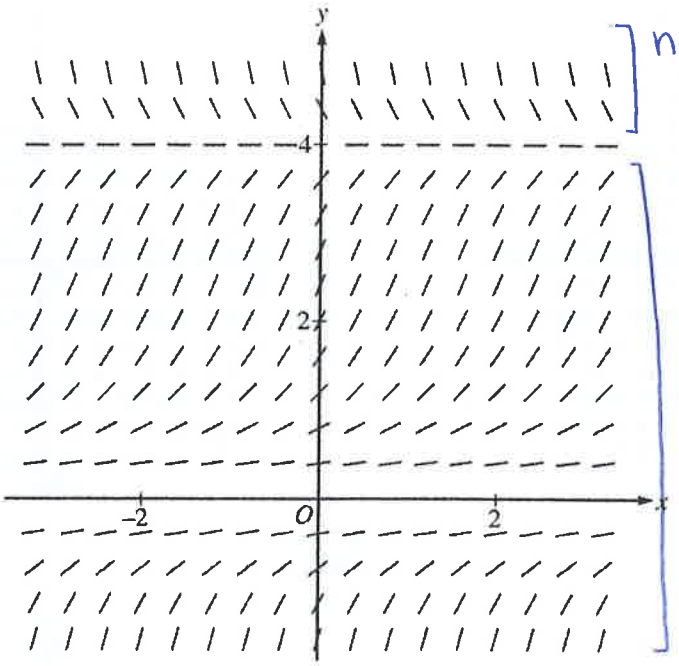
Exam 11
Differential and Slope Field
MC Review Packet
No Calculator

Name Teacher Copy



notice $y > 4$ slope is negative

notice $y < 4$ all slopes positive

13. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x(4-y)}{4}$

(B) $\frac{dy}{dx} = \frac{y(4-y)}{4}$

(C) $\frac{dy}{dx} = \frac{xy(4-y)}{4}$

(D) $\frac{dy}{dx} = \frac{y^2(4-y)}{4}$

14. The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$

(B) $120e^{t/5}$

(C) $e^{t/5} + 119$

(D) $24t + 120$

$\frac{dy}{dt} = \frac{1}{5}y$

$\frac{1}{y}dy = \frac{1}{5}dt$

$\ln|y| = \frac{1}{5}t + C$

$|y| = e^{\frac{1}{5}t + C}$

$|y| = e^C \cdot e^{\frac{1}{5}t}$

$y = \pm e^C \cdot e^{\frac{1}{5}t}$

Let $k_2 = \pm e^C$

$\frac{dy}{dt} = 24$

$24 = 120k$

$\frac{24}{120} = k$

$\frac{1}{5} = k$

$y = k_2 e^{\frac{1}{5}t}$

$120 = k_2 e^0$

$k_2 = 120$

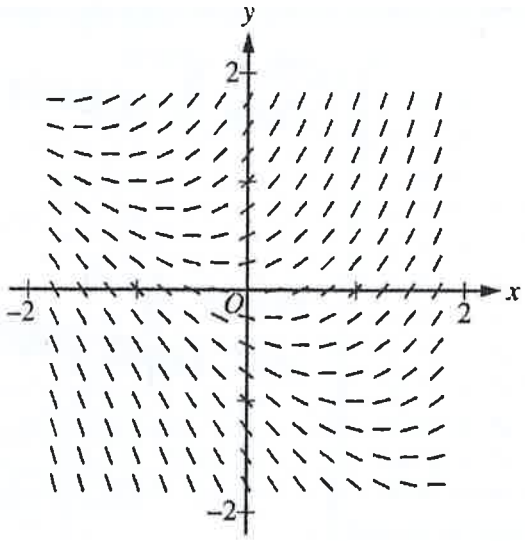
$y = 120e^{\frac{1}{5}t}$

(#, +) = pos. slope

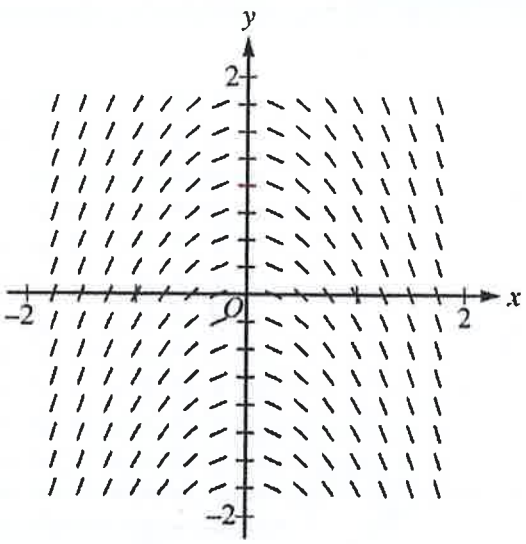
16. Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

↖ any x will be positive

(A)



(B)



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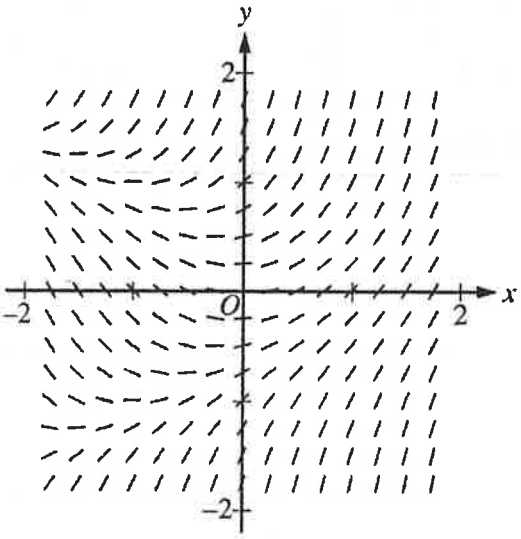

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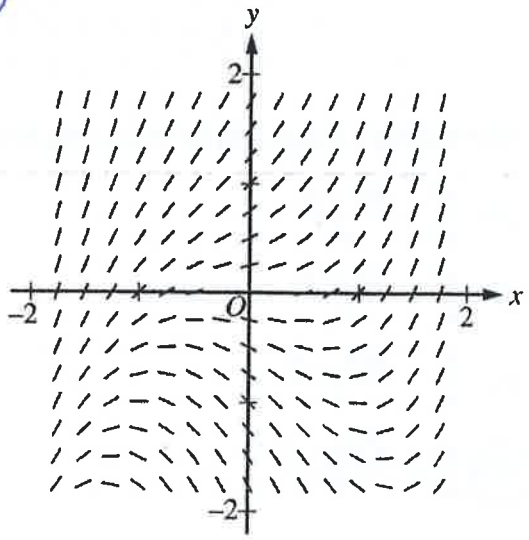


22. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with the initial condition $f(0) = -2$. Which of the following is an expression for $f(x)$?

(A) $-2 - \sqrt{x^2 + 2x}$

(B) $-2 + \sqrt{x^2 + 2x}$

(C) $-\sqrt{x^2 + 2x + 4}$

(D) $\sqrt{x^2 + 2x + 4}$

$y\,dy = (x+1)\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$\frac{(-2)^2}{2} = 0 + 0 + C$

$C = 2$

$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$

$y^2 = x^2 + 2x + 4$

$y = \pm\sqrt{x^2 + 2x + 4}$

check $-2 = \pm\sqrt{0+0+4}$

14. If $y = f(x)$ is a solution to the differential equation $\frac{dy}{dx} = e^{x^2}$ with the initial condition $f(0) = 2$, which of the following is true?

*work backwards

(A) $f(x) = 1 + e^{x^2}$ $f'(x) = e^{x^2} \cdot 2x$

(B) $f(x) = 2xe^{x^2}$ $f(0) = 2(0)e^0 = 0$

(C) $f(x) = \int_1^x e^{t^2}\,dt$ doesn't consider initial condition

(D) $f(x) = 2 + \int_0^x e^{t^2}\,dt$

(E) $f(x) = 2 + \int_2^x e^{t^2}\,dt$ ↖ not initial condition

18. A student attempted to solve the differential equation $\frac{dy}{dx} = xy$ with initial condition $y = 2$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y}\,dy = \int x\,dx$

Step 2: $\ln|y| = \frac{x^2}{2} + C$

Step 3: $|y| = e^{x^2/2} + C$ ↙ e^C

Step 4: Since $y = 2$ when $x = 0$, $2 = e^0 + C$.

Step 5: $y = e^{x^2/2} + 1$

(A) Step 2
(B) Step 3
(C) Step 4
(D) Step 5
(E) There is no error in the solution.

3. Let f be a differentiable function such that $f(2) = 4$ and $f'(2) = -\frac{1}{2}$. What is the approximation for $f(2.1)$ found by using the line tangent to the graph of f at $x = 2$?

↖ means slope at $x = 2$

(A) 2.95 (B) 3.95 (C) 4.05 (D) 4.1

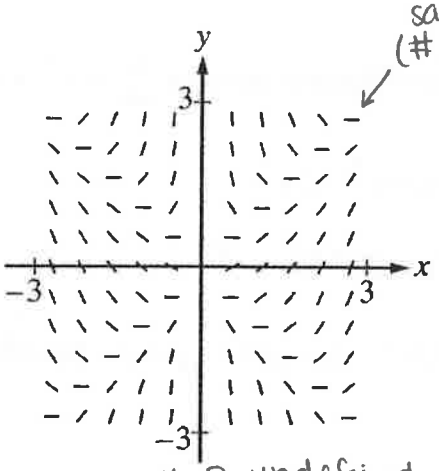
$y - 4 = -\frac{1}{2}(x-2)$

$f(2.1)$ $y - 4 = -\frac{1}{2}(2.1-2)$

$y - 4 = -.5(0.1)$

$y - 4 = -.05$

$y = 3.95$



same (#,#) gives m=0

x=0 undefined

13. Shown above is a slope field for which of the following differential equations?

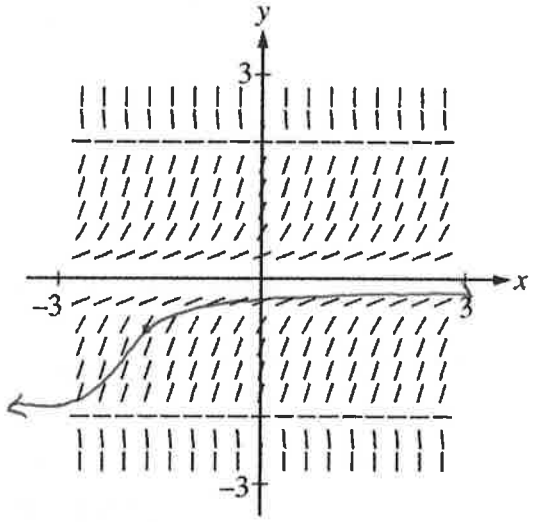
(A) $\frac{dy}{dx} = \frac{x^2 - y^2}{x}$

(B) $\frac{dy}{dx} = \frac{x^2 - y^2}{y}$

(C) $\frac{dy}{dx} = x^2 - y^2$

(D) $\frac{dy}{dx} = \frac{x^2 + y^2}{x}$

(E) $\frac{dy}{dx} = x^2 + y^2$



8. Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2(4 - y^2)$. If $y = g(x)$ is the solution to the differential equation with the initial condition $g(-2) = -1$, then $\lim_{x \to \infty} g(x)$ is

(A) $-\infty$ (B) -2 (C) 0 (D) 2 (E) 3

↖ horizontal asymptote

24. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 5y^2$ with the initial condition $y(0) = 3$?

(A) $y = \sqrt{9e^{5x}}$

(B) $y = \sqrt{\frac{1}{9}e^{5x}}$

(C) $y = \sqrt{e^{5x} + 9}$

(D) $y = \frac{3}{1 - 15x}$

(E) $y = \frac{3}{1 + 15x}$

$\frac{1}{y^2}dy = 5\,dx$

$\int y^{-2}dy = \int 5\,dx$

$\frac{-1}{y} = 5x + C$

$\frac{-1}{3} = 5(0) + C$

$C = \frac{-1}{3}$

$\frac{-1}{y} = 5x - \frac{1}{3}$

$-1 = y(5x - \frac{1}{3})$

$\frac{-1}{5x - \frac{1}{3}} = y$

$\frac{-3}{15x - 1} = y$

$\frac{3}{1 - 15x} = y$

23. What is the particular solution to the differential equation $\frac{dy}{dx} = xy^2$ with the initial condition $y(2) = 1$?

(A) $y = e^{\frac{x^2}{2} - 2}$

(B) $y = e^{\frac{x^2}{2}}$

(C) $y = -\frac{2}{x^2}$

(D) $y = \frac{2}{6 - x^2}$

(E) $y = \frac{6 - x^2}{2}$

$\frac{1}{y^2}dy = x\,dx$

$\int y^{-2}dy = \int x\,dx$

$\frac{-1}{y} = \frac{x^2}{2} + C$

$-1 = 2 + C$

$C = -3$

$\frac{-1}{y} = \frac{x^2}{2} - 3$

$-1 = (\frac{x^2}{2} - 3)y$

$y = \frac{-1}{\frac{x^2}{2} - 3}$

$y = \frac{-2}{x^2 - 6}$ or $\frac{2}{6 - x^2}$

24. Which of the following is the solution to the differential equation $\frac{dy}{dx} = -2xy$ with the initial condition $y(1) = 4$?

(A) $y = e^{x^2} + 4 - e$

(B) $y = e^{-x^2} + 4 - \frac{1}{e}$

(C) $y = 4e^{x^2 - 1}$

(D) $y = 4e^{-x^2 + 1}$

(E) $y = e^{-x^2 + 16}$

$\frac{1}{y}dy = -2x\,dx$

$\ln|y| = -x^2 + C$

$|y| = e^{-x^2} \cdot e^C$

$y = \pm e^C \cdot e^{-x^2}$

Let $K = \pm e^C$

$y = Ke^{-x^2}$

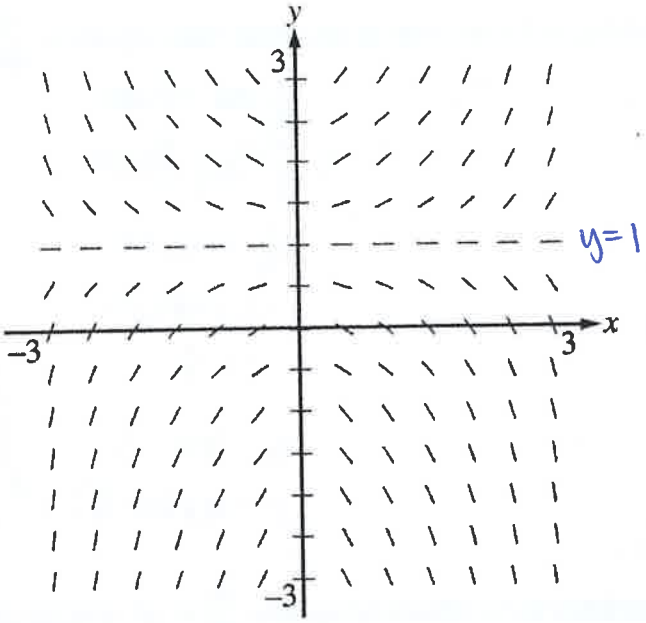
$4 = Ke^{-1}$

$4 = \frac{K}{e}$

$4e = K$

$y = 4e \cdot e^{-x^2}$

$y = 4e^{-x^2 + 1}$



28. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy - x$

(B) $\frac{dy}{dx} = xy + x$

(C) $\frac{dy}{dx} = y - x^2$

(D) $\frac{dy}{dx} = (y-1)x^2$

(E) $\frac{dy}{dx} = (y-1)^3$

13. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$ $3y^2\frac{dy}{dx} + \frac{dy}{dx} = 2x$ $\frac{dy}{dx}(3y^2+1) = 2x$ $\frac{dy}{dx} = \frac{2x}{3y^2+1}$

(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1+3y^2}$

18. If $\ln(2x + y) = x + 1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) $y - \frac{y}{x}$

$\frac{1}{2x+y}\left(2+\frac{dy}{dx}\right) = 1$

$2 + \frac{dy}{dx} = 2x+y$

$\frac{dy}{dx} = 2x+y-2$

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln(-e^x + 5)$

(D) $y = -\ln(e^x + 3)$

(E) $y = \ln(e^x + 3)$

$\frac{dy}{dx} = e^y \cdot e^x$

$\frac{1}{e^y}\,dy = e^x\,dx$

$\int e^{-y}\,dy = \int e^x\,dx$

$-e^{-y} = e^x + C$

$-e^{\ln 4} = 1 + C$

$-4 = 1 + C$

$C = -5$

$-e^{-y} = e^x - 5$

$e^{-y} = -e^x + 5$

$-y = \ln(-e^x + 5)$

$y = -\ln(-e^x + 5)$

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

$y = mx + b$

$y' = m$

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

$dy = 2\sin x\,dx$

$y = -2\cos x + C$

$1 = -2\cos\pi + C$

$1 = -2(-1) + C$

$1 = 2 + C$

$-1 = C$

$y = -2\cos x - 1$

$\frac{dy}{dx} = \frac{2x-y}{x+2y}$ $\quad \frac{2(3)-0}{3+2(0)} = \frac{6}{3} = 2$

27. If $(x+2y)\cdot\frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$

$= \frac{(3+0)(2-2) - (6-0)(1+4)}{(3+0)^2}$

$= \frac{(3)(0)-(6)(5)}{9} = \frac{-30}{9} = \frac{-10}{3}$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

$\cos(xy)\left[x\frac{dy}{dx}+y\right] = 1$

$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 1$

$x\cos(xy)\frac{dy}{dx} = 1 - y\cos(xy)$

$\frac{dy}{dx} = \frac{1-y\cos(xy)}{x\cos(xy)}$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

$y = -x + k \quad m = -1$

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

$y' = 2x + 3$

$-1 = 2x + 3$

$-4 = 2x$

$-2 = x$

$y = (-2)^2 + 3(-2) + 1$

$= 4 - 6 + 1$

$= -1$

$(-2, -1)$ $\quad y = -x + k$

$-1 = 2 + k$

$-3 = k$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

$y\,dy = x^2\,dx$

$\frac{y^2}{2} = \frac{x^3}{3} + C$

$\frac{(-2)^2}{2} = \frac{3^3}{3} + C$

$2 = 9 + C$

$C = -7$

$\frac{y^2}{2} = \frac{x^3}{3} - 7$

$y^2 = \frac{2x^3}{3} - 14$

$y = \pm\sqrt{\frac{2x^3}{3} - 14}$

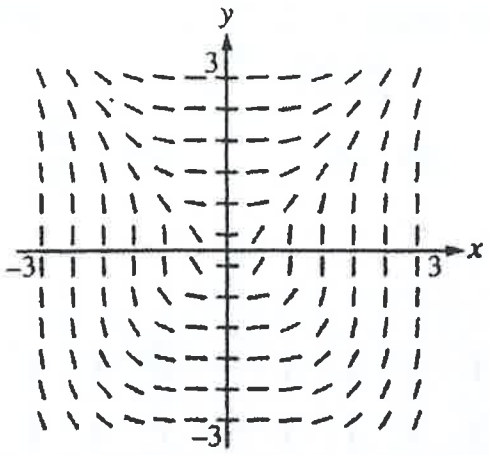
(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$



14. **Shown above is a slope field for which of the following differential equations?**

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

check Quad 3 $\frac{-}{-} = +$ m= all +

18. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

$dy = -10e^{-t/2}\,dt$

$u = \frac{-t}{2}$

$du = \frac{-1}{2}dt$

$-2du = dt$

$\int dy = \int 20e^{u}\,du$

$y = 20e^{\frac{-t}{2}} + C$

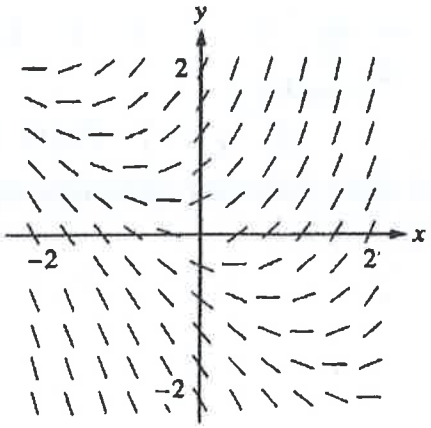
$20 = 20e^{0} + C$

$C = 0$

$y = 20e^{\frac{-t}{2}}$

$y(6) = 20e^{\frac{-6}{2}}$

$20e^{-3}$



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

all positive slope

Quad 3

↑ can't be negative

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

$\frac{1}{y}dy = k\,dt$

$\ln|y| = kt + C$

$y = \pm e^{C} \cdot e^{kt}$

constant

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$2x + x\frac{dy}{dx} + y = 0$

$x\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx} = \frac{-2x-y}{x}$

$(2)^2 + (2)y = 10$

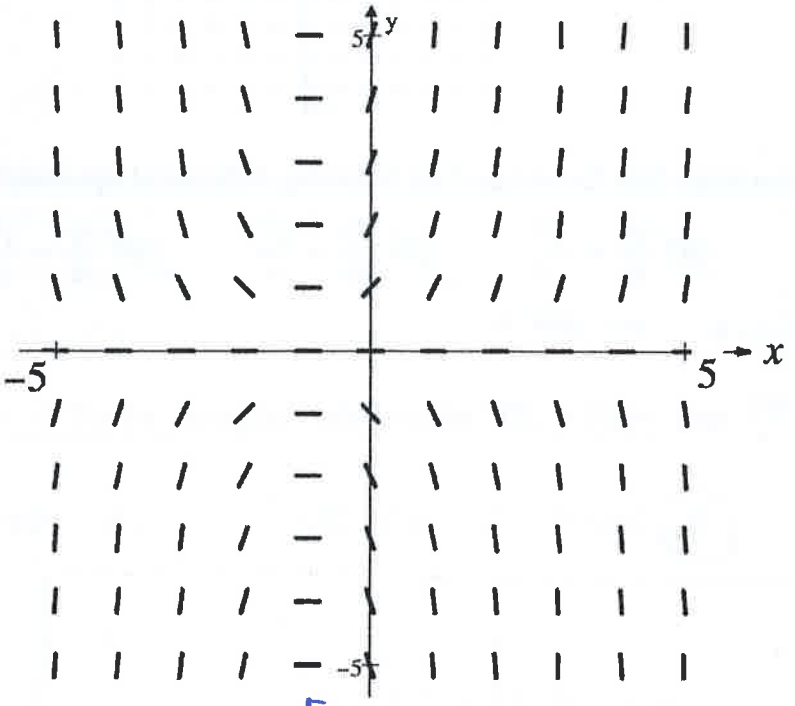
$2y = 6$

$y = 3$

(2, 3)

$\frac{-2(2)-3}{2} = -\frac{7}{2}$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$



notice if $x = -1$ then $m = 0$

27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

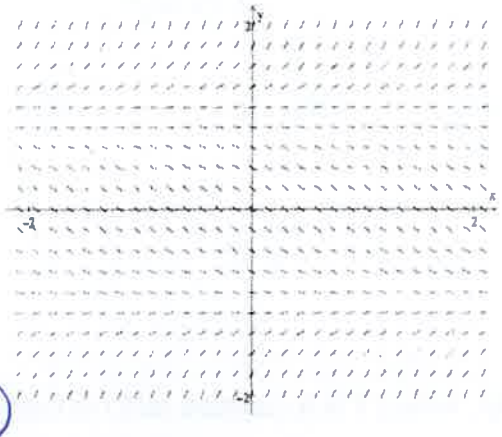
(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

since only y in the equation y=# should have the same slope across the graph

(A)



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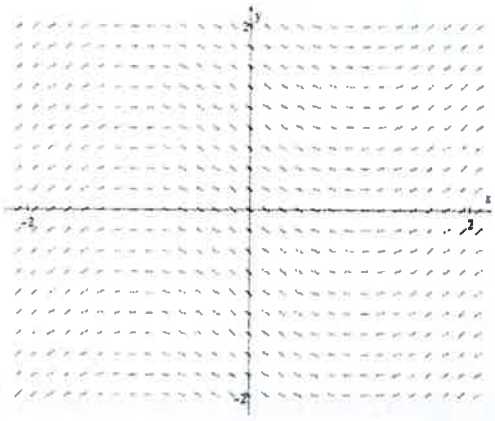

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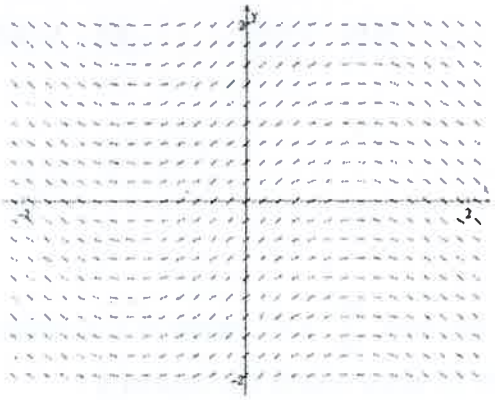

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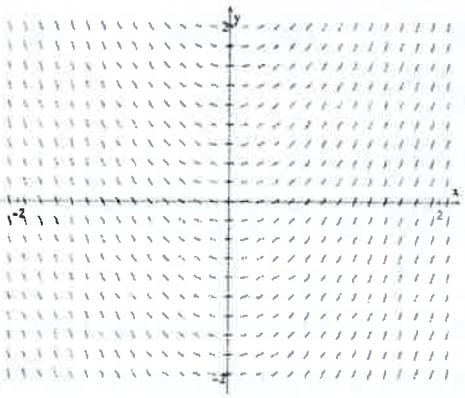

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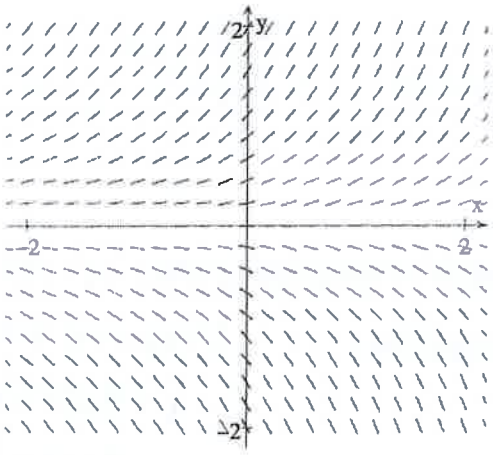

81. If $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$ $\frac{1}{2}(1-y^2)^{-1/2} \cdot (-2y\frac{dy}{dx}) = \frac{-2y\sqrt{1-y^2}}{2\sqrt{1-y^2}}$

$(1-y^2)^{\frac{1}{2}}$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

$2 = e^{10k}$

$\ln 2 = 10k$

$k = \frac{\ln 2}{10} \approx 0.0693147$

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k?

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

$2 = e^{12k}$

$\ln 2 = 12k$

$k = \frac{\ln 2}{12} \approx 0.0577622665$

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87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

$y - 0.115226 = 1(x - 0.236733)$

$y = x - 0.121507$

$f'(x) = 4x^3 + 4x$

$1 = 4x^3 + 4x$

$4x^3 + 4x - 1 = 0$

then use calc to solve

$x = 0.236733$

$f(0.236733) = 0.115226$