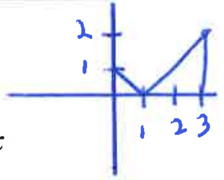
AP Calculus AB

Mixed Integral W.S.

Name Key

Evaluate

1. $\int_0^3 \sqrt{x^2-2x+1}\,dx$



$\frac{1}{2}\cdot 1\cdot 1+\frac{1}{2}\cdot 2\cdot 2 = \boxed{\frac{5}{2}}$

or $\int_0^3 |x-1|\,dx = -\int_0^1 (x-1)\,dx + \int_1^3 (x-1)\,dx$

$= -(\frac{1}{2}x^2-x)\Big|_0^1 + (\frac{1}{2}x^2-x)\Big|_1^3$

$= -(\frac{1}{2}\cdot 1^2-1) + (\frac{1}{2}\cdot 3^2-3) - (\frac{1}{2}\cdot 1^2-1)$

$= \frac{1}{2}+\frac{9}{2}-\frac{6}{2}+\frac{1}{2} = \boxed{\frac{5}{2}}$

2. $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}\,dx = \int_0^{\pi/4} \tan x\cdot\sec x\,dx = \sec x\Big|_0^{\pi/4}$

$= \sec\frac{\pi}{4} - \sec 0$

$= \boxed{\sqrt{2}-1}$

3. $\int_0^1 \frac{5}{1+x^2}\,dx$

$5\int_0^1 \frac{1}{1+x^2}\,dx = 5\tan^{-1}(x)\Big|_0^1$

$= 5[\tan^{-1}(1)-\tan^{-1}(0)]$

$= 5[\frac{\pi}{4}-0] = \boxed{\frac{5\pi}{4}}$

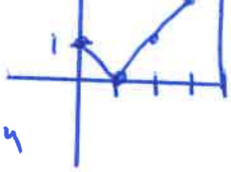
4. $\int_1^9 x\sqrt{3x}\,dx = \int_1^9 x\cdot\sqrt{3}\cdot x^{1/2}\,dx = \sqrt{3}\int_1^9 x^{3/2}\,dx$

$= \sqrt{3}\cdot\frac{2}{5}x^{5/2}\Big|_1^9 = \frac{2\sqrt{3}}{5}[9^{5/2}-1^{5/2}]$

$= \frac{2\sqrt{3}}{5}[243-1] = \boxed{\frac{484\sqrt{3}}{5}}$

5. $\int_0^4 |x-1|\,dx$

$\frac{1}{2}\cdot 1\cdot 1+\frac{1}{2}\cdot 3\cdot 3 = \boxed{5}$



$= -\int_0^1 (x-1)\,dx + \int_1^4 (x-1)\,dx$

$-(\frac{1}{2}x^2-x)\Big|_0^1 + (\frac{1}{2}x^2-x)\Big|_1^4$

$-(\frac{1}{2}\cdot 1^2-1) - -(0) + (\frac{1}{2}(4^2)-4) - (\frac{1}{2}\cdot 1^2-1)$

$\frac{1}{2}+4+1/2 = \boxed{5}$

6. $\int_{-2}^5 \frac{5}{3x}\,dx$ No sol - undefined at $x=0$

7. $\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{1/2}(x+1)\,dx = \int_0^1 x^{3/2}+x^{1/2}\,dx$

$= \frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}\Big|_0^1 = (\frac{2}{5}+\frac{2}{3}) = \boxed{\frac{16}{15}}$

8. $\int_1^e \frac{x^2-1}{x}\,dx = \int_1^e (x-\frac{1}{x})\,dx = \frac{1}{2}x^2-\ln|x|\Big|_1^e$

$= (\frac{1}{2}e^2-\ln e) - (\frac{1}{2}\cdot 1^2-\ln 1)$

$= \frac{1}{2}e^2-1-1/2 = \boxed{\frac{1}{2}e^2-\frac{3}{2} = \frac{e^2-3}{2}}$

9. $\int_{-2}^3 |x^2-x-6|\,dx$

$(x-3)(x+2)$

+ − +
-2 3

$-\int_{-2}^3 (x^2-x-6)\,dx = -(\frac{1}{3}x^3-\frac{1}{2}x^2-6x)\Big|_{-2}^3$

$= -(\frac{1}{3}\cdot 3^3-\frac{1}{2}\cdot 3^2-6\cdot 3) - -(\frac{1}{3}(-2)^3-\frac{1}{2}(-2)^2-6(-2))$

$= -(9-\frac{9}{2}-18)+(-\frac{8}{3}-2+12) = \boxed{\frac{125}{6}}$

10. $\int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x\Big|_{\sqrt{2}/2}^1 =$

$\sin^{-1}(1)-\sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{2}-\frac{\pi}{4} = \boxed{\frac{\pi}{4}}$

11. $\int_{-1}^4 |-x^2+2x|\,dx = -\int_0^1 (-x^2+2x)\,dx + \int_0^2 (-x^2+2x)\,dx + -\int_2^4 (-x^2+2x)\,dx$

$-x(x-2)$

− + −
0 2

$= -(\frac{1}{3}x^3+x^2)\Big|_0^1 + (\frac{1}{3}x^3+x^2\Big|_0^2 + -(\frac{1}{3}x^3+x^2)\Big|_2^4 = \boxed{\frac{28}{3}}$

12. $\int_0^4 \left(2+\sqrt{16-x^2}\right)dx$

$\int_0^4 2\,dx + \int_0^4 \sqrt{16-x^2}\,dx$

$2x\Big|_0^4 + \frac{1}{4}\pi\cdot 4^2$

$= \boxed{8+4\pi}$

Find the derivative

13. $g(x) = \int_0^{\sin \pi x} \sqrt{t+3t^2}\,dt$

$g'(x) = \sqrt{\sin\pi x + 3\sin^2\pi x} \cdot (\sin \pi x)'$

$= \sqrt{\sin\pi x + 3\sin^2\pi x} \cdot \pi\cos(\pi x)$

14. $h(x) = \int_x^{x^3} \cos(t)\,dt$

$= \int_{\cancel{x}}^{0} \cos t\,dt + \int_0^{x^3} \cos t\,dt$

$= -\int_0^{x} \cos t\,dt + \int_0^{x^3} \cos t\,dt$

$-\cos x + 3x^2 \cos x^3$

15. $\frac{d}{dx}\int_1^{e^x} \frac{1}{t}\,dt = \ln t \Big|_1^{e^x} = \ln e^x - \ln 1$

$= x \ln e - 0$

$= x$

$\frac{d}{dx}(x) = 1$

FTC

$\frac{1}{e^x} \cdot e^x = 1$

16. $\frac{d}{dx}\int_2^{x^2} (t^2-5)dt$

$((x^2)^2 - 5) \cdot \frac{d}{dx} x^2$

$2x(x^4 - 5) = 2x^5 - 10x$

17. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

$F'(x) = \sqrt{x^3+1}$

$F'(2) = \sqrt{2^3+1} = \sqrt{9} = 3$

18. If $F(x) = \int_1^{x^2} \ln t\,dt$, then $F''(x) =$

$F'(x) = \ln x^2 \cdot 2x = 2x \ln x^2$

$F''(x) = (2x)' \ln x^2 + (\ln x^2)'(2x)$

$= 2\ln x^2 + \frac{1}{x^2} \cdot 2x \cdot 2x$

$\boxed{2\ln x^2 + 4}$

19. If $F(x) = \int_2^x (t^2+2t-15)\,dt$,

then $F(x)$ is increasing on what interval(s)?

$F'(x) = x^2 + 2x - 15$

$F'(x) = 0 = (x+5)(x-3)$

$x = -5 \quad x = 3$

$F'(x)$ + | − | +

$F(x)$ inc 5 3 inc

$\boxed{(-\infty, -5) \cup (3, \infty)}$

20. If $F(x) = \int_3^x \frac{t-2}{t}\,dt$,

then $F(x)$ is decreasing on what interval(s)?

$F'(x) = \frac{x-2}{x}$

$F'(x)$ + | − | +

$F(x)$ inc 0 de 2 inc

$\boxed{(0, 2)}$

21. If $f(x) = \begin{cases} 8-x^2 & for\ -2 \le x \le 2 \\ x^2 & elsewhere \end{cases}$, then $\int_{-1}^{3} f(x)\,dx$ is?

$\int_{-1}^{2} (8-x^2)\,dx + \int_2^3 x^2\,dx$

$8x - \frac{1}{3}x^3 \Big|_{-1}^{2} + \frac{1}{3}x^3 \Big|_2^3$

$(8 \cdot 2 - \frac{1}{3} \cdot 2^3) - (8 \cdot -1 - \frac{1}{3}(-1)^3) + (\frac{1}{3} \cdot 3^3 - \frac{1}{3} 2^3)$

$16 - \frac{8}{3} - (-8 + 1/3) + (9 - \frac{8}{3}) = \frac{82}{3}$

22. Evaluate $\int_0^6 f(x)dx$, if $f(x) = \begin{cases} x^2 & , x \le 2 \\ 3x-2, & x \ge 2 \end{cases}$

$\int_0^2 x^2\,dx + \int_2^6 3x - 2\,dx$

$\frac{1}{3}x^3 \Big|_0^2 + \frac{3}{2}x^2 - 2x \Big|_2^6$

$\frac{1}{3}(2)^3 - 0 + (\frac{3}{2}(6)^2 - 2(6)) - (\frac{3}{2} \cdot 2^2 - 2 \cdot 2)$

$\frac{8}{3} + 54 - 12 - 6 + 4 = \frac{128}{3}$

23. If $f(x)=\begin{cases} x+1 \text{ for } x<0, \\ \cos(x) \text{ for } x\geq 0 \end{cases}$, find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx$

$\int_{-\frac{\pi}{2}}^{0} (x+1)\,dx + \int_{0}^{\pi/2} \cos x\,dx$

$\left(\frac{x^2}{2}+x\right)\Big|_{-\frac{\pi}{2}}^{0} + (\sin x)\Big|_{0}^{\pi/2}$

$\left[0-\left(\frac{\pi^2}{8}-\frac{\pi}{2}\right)\right]+\left[\sin\frac{\pi}{2}-\cancel{\sin 0}\right] = \boxed{-\frac{\pi^2}{8}+\frac{\pi}{2}+1}$

24. If $f(x)=x|x|\,dx$, find. $\int_{-2}^{3} f(x)\,dx$

$\begin{cases} -x^2 & x\leq 0 \\ x^2 & x\geq 0 \end{cases}$

$\int_{-2}^{0} -x^2\,dx + \int_{0}^{3} x^2\,dx$

$-\frac{x^3}{3}\Big|_{-2}^{0} + \frac{x^3}{3}\Big|_{0}^{3}$

$\left(0-\frac{8}{3}\right)+\left(\frac{27}{3}-0\right)$

$-\frac{8}{3}+\frac{27}{3} = \boxed{\frac{19}{3}}$

Find the average value.

25. $3x^3-x^2$ *on* $[-1,2]$

$\frac{1}{2-(-1)}\int_{1}^{2}(3x^3-x^2)\,dx$

$\frac{1}{3}\left(\frac{3x^4}{4}-\frac{x^3}{3}\right)\Big|_{-1}^{2}$

$\frac{1}{3}\left[\left(\frac{48}{4}-\frac{8}{3}\right)-\left(\frac{3}{4}+\frac{1}{3}\right)\right]$

$\rightarrow \frac{1}{3}\left[\frac{28}{3}-\frac{13}{12}\right]$

$\frac{1}{3}\left(\frac{33}{4}\right) = \boxed{\frac{11}{4}}$

26. $\sqrt{x}$ *on* $[0,2]$

$\frac{1}{2-0}\int_{0}^{2}\sqrt{x}\,dx$

$\frac{1}{2}\left(\frac{2}{3}x^{3/2}\right)\Big|_{0}^{2} = \frac{1}{3}\left[2^{3/2}-0\right] = \boxed{\frac{2\sqrt{2}}{3}}$

27. $\sqrt{3x}$ on $[0,9]$

$\frac{1}{9-0}\int_{0}^{9}\sqrt{3x}\,dx = \frac{1}{9}\sqrt{3}\int_{0}^{9}\sqrt{x}\,dx$

$\frac{\sqrt{3}}{9}\left(\frac{2}{3}x^{3/2}\right)\Big|_{0}^{9} = \frac{2\sqrt{3}}{27}\left(9^{3/2}-0\right) = \frac{2\sqrt{3}}{27}(27) = \boxed{2\sqrt{3}}$

28. $\frac{1}{x}$ on $[1,e]$

$\frac{1}{e-1}\int_{1}^{e}\frac{1}{x}\,dx = \frac{1}{e-1}\ln x\Big|_{1}^{e}$

$\frac{1}{e-1}(\ln e-\ln 1) = \frac{1}{e-1}(1-0) = \boxed{\frac{1}{e-1}}$

29. The average value of $2x+1$ on $[a,4]$ is 5.

Find the value of a

$\frac{1}{4-a}\int_{a}^{4}(2x+1)\,dx = 5$

$(x^2+x)\Big|_{a}^{4} = 5(4-a)$

$(16+4)-(a^2+a) = 20-5a$

$-a^2-a+20 = 20-5a$

$\rightarrow a^2-4a=0$

$a(a-4)=0$

$\boxed{a=0}, 4$

30. The average value of a cont. function $f(x)$ on the closed interval [3,7] is 12. What is the value of $\int_{3}^{7} f(x)\,dx$?

$\frac{1}{7-3}\int_{3}^{7} f(x)\,dx = 12$

$\frac{1}{4}\int_{3}^{7} f(x)\,dx = 12$

$\int_{3}^{7} f(x)\,dx = \boxed{48}$

Evaluate

31. If $\int_{-2}^{2}(x^7+k)\,dx=16$, *then* $k=$

$\left(\frac{x^8}{8}+kx\right)\Big|_{-2}^{2} = 16$

$\left(\frac{256}{8}+2k\right)-\left(\frac{256}{8}-2k\right) = 16$

$2k+2k=16$

$4k=16$

$\boxed{k=4}$

32. If $\int_{2}^{7} f(x)\,dx=6$ then $\int_{2}^{7}(2f(x)+3)\,dx=?$

$2\int_{2}^{7} f(x)\,dx + \int_{2}^{7} 3\,dx$

$2(6) + 3x\Big|_{2}^{7}$

$12+(21-6)$

$12+15 = \boxed{27}$

33. Given $\int_{1}^{7} f'(x)\,dx=17$, *and* $f(7)=7$ find $f(1)$

$\int_{1}^{7} f'(x)\,dx = f(x)\Big|_{1}^{7} = f(7)-f(1)$

$7-f(1)=17$

$-f(1)=10$

$\boxed{f(1)=-10}$

34. Given $\int_{2}^{15} f'(x)\,dx=20$ and $f(2)=6$, find $f(15)$

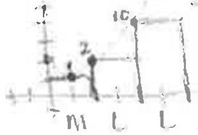
$\int_{2}^{15} f'(x)\,dx = f(x)\Big|_{2}^{15} = f(15)-f(2)$

$f(15)-6=20$

$\boxed{f(15)=26}$

vertex: $x = \frac{-(-2)}{2(1)} = 1$ $y = 1^2 - 2 + 2$ $\Delta x = \frac{6}{3} = 2$

35. If $\int_0^6 \left(x^2 - 2x + 2\right) dx$ is approximated by three **inscribed** rectangles of equal width on the x-axis, then the approximation is ? (Hint: sketch and make sure each rectangle is inside the curve)



a. 24   [✓ b. 26]   c. 28   d. 48   e. 76

$(1 \cdot 2) + (2 \cdot 2) + (10 \cdot 2) = (2 + 4 + 20)$

36. Let f be the function given by $f(x) = 3 + \int_0^x \cos\left(t^2\right) dt$. What is the least positive number a, for which $f'(a) = 0$? (use calculator, graph and use calculate zero)

$\cos(x^2) = 0$

a. 1   [✓ b. 1.253]   c. 1.571   d. 1.772   e. 3.142

37.

x	-2	-1	0	1	2
$f(x)$	a	b	c	d	e
$f'(x)$	2	4	6	8	10

$\frac{1}{2+2} \int_{-2}^{2} f'(x)\,dx$

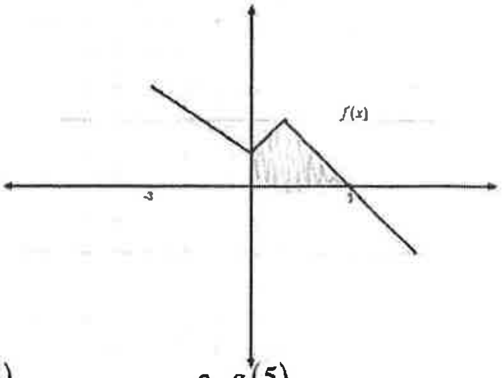
$\frac{1}{4}\left[f(2) - f(-2)\right] = \frac{1}{4}\left[e - a\right] = \frac{e-a}{4}$

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on $[-2, 2]$, what is the average value of $f'(x)$ on the closed interval $[-2, 2]$?

a. $e - a$ [✓ b. $\frac{e-a}{4}$]   c. $\frac{a-e}{4}$ d. $\frac{a-e}{8}$ e. $\frac{e-a}{8}$

38. The graph of a piecewise function f is shown to the right. If $g(x) = \int_0^x f(t)\,dt$, which of the following has the greatest value?

positive area



a. $g(-3)$ b. $g(0)$ c. $g(1)$ [✓ d. $g(3)$]   e. $g(5)$

39. $\frac{d}{dx} \int_x^{x^2} \sin\left(t^2\right) dt$

$\sin(x^2)^2 \cdot 2x - \sin x^2 \cdot 1$

$\boxed{2x \sin(x^4) - \sin x^2}$

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40. A tank is being filled with water at the rate of $300\sqrt{t}$ gallons per hour with $t > 0$ measured in hours. If the tank is originally empty, how many gallons of water are in the tank after 4 hours?

$\int_0^4 300\sqrt{t}\,dt = 300\left(\frac{2}{3}t^{3/2}\right)\Big|_0^4$

$200\left(4^{3/2}-0\right) = 200(2^3) = 1600$

a. 600 b. 900 c. 1200 [✓ d. 1600] e. 2400

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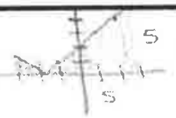
41. $\int_{-3}^{3}|x+2|\,dx =$



$= \frac{1}{2}(1)(1) + \frac{1}{2}(5)(5)$

$\frac{1}{2} + \frac{25}{2} = \frac{26}{2} = 13$

a. 0 b. 8 [✓ c. 13] d. 17 e. 21

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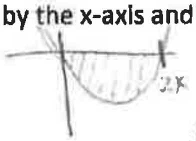
42. Let R be the region in the fourth quadrant enclosed by the x-axis and the curve $y = x^2 - 2kx$, where k is a constant. If the area of the region R is 36, then the value of k is



$x(x-2k)$ $A = -36$

$x = 0, 2k$

a. -3 [✓ b. 3] c. 4 d. 6 e. $\pm 3\sqrt[3]{2}$

$\int_0^{2k}(x^2-2kx)\,dx = -36$ $\frac{x^3}{3} - kx^2\Big|_0^{2k} = -36$ $\left[\frac{(2k)^3}{3} - k(2k)^2\right] - 0 = -36$

$\frac{8k^3}{3} - 4k^3 = -36$

$8k^3 - 12k^3 = -108$

$-4k^3 = -108$

$k^3 = 27$

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43. If $f(x) = 15 - g(x)$ for $-2 \le x \le 2$, then $\int_{-2}^{2}[f(x) - g(x)]dx =$

$\int_{-2}^{2} 15 - 2g(x)\,dx = \int_{-2}^{2} 15\,dx - 2\int_{-2}^{2} g(x)\,dx$

$15x\Big|_{-2}^{2} - 2\int_{-2}^{2} g(x)\,dx = (30+30) - 2\int_{-2}^{2} g(x)\,dx$

a. 60 b. $2\int_{-2}^{2} g(x)\,dx - 60$ c. $2\int_{-2}^{2} g(x)\,dx + 60$ d. $60 - 4\int_{-2}^{2} g(x)\,dx$ [✓ e. $60 - 2\int_{-2}^{2} g(x)\,dx$]

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44. $\int_2^4\left[\frac{d}{dx}(3t^2 + 2t - 1)\right]dt$

$\int_2^4 6t + 2\,dt$

$(3t^2 + 2t)\Big|_2^4$

$(48+8) - (12+4)$

$56 - 16 = 40$

a. 12 [✓ b. 40] c. 46 d. 55 e. 66

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45. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)dx = 6$, then $\int_8^4 f(x)dx =$

$\int_2^4 f(x)dx + \int_4^8 f(x)dx = \int_2^8 f(x)dx$

$6 + \int_4^8 f(x)dx = -10$

a. -16 b. -6 c. -4 d. 4 [✓ e. 16]

$\int_4^8 f(x)dx = -16$

$-\int_8^4 f(x)dx = -16$

$\int_8^4 f(x)dx = 16$

Mixed Integral W.S.
Answers

1. $\frac{5}{2}$
2. $\sqrt{2}-1$
3. $\frac{5\pi}{4}$
4. $\frac{484\sqrt{3}}{5}$
5. 5

6. no sol. (discount.)
7. $\frac{16}{15}$
8. $\frac{e^2-3}{2}$
9. $\frac{125}{6}$
10. $\frac{\pi}{4}$

11. $\frac{28}{3}$
12. $8+4\pi$
13. $\pi\cos(\pi x)\sqrt{\sin(\pi x)+3\sin^2(\pi x)}$

14. $3x^2\cos\left(x^3\right)-\cos x$
15. 1
16. $2x\left(x^4-5\right)$
17. 3
18. $4+2\ln x^2$

19. $(-\infty,-5)\cup(3,\infty)$
20. $(0,2)$
21. $\frac{82}{3}$
22. $\frac{128}{3}$

23. $-\frac{\pi^2}{8}+\frac{\pi}{2}+1$
24. $\frac{19}{3}$
25. $\frac{11}{4}$
26. $\frac{2\sqrt{2}}{3}$

27. $2\sqrt{3}$
28. $\frac{1}{e-1}$
29. $a=0$ *only*
30. 48

31. $k=4$
32. 27
33. $f(1)=-10$
34. $f(15)=26$

35. b
36. b
37. b
38. d
39. $2x\sin x^4-\sin x^2$

40. d
41. c
42. b
43. e
44. b
45. e