

**Riemann Sums/Trapezoidal Rule
Worksheet**

Name _____

Estimate each of the following:

1. $\int_2^6 (3x+1) dx$ using L_4

2. $\int_0^{16} (x^2+1) dx$ using R_4

3. $\int_1^9 (2^x-1) dx$ using M_2

4. $\int_1^{15} \ln x dx$ using L_2 (calculator, round to three decimals)

5. $\int_2^{18} \frac{4}{x} dx$ using L_4 (calculator, three decimals places)

6. $\int_1^3 (2x-1) dx$ using T_2

7. $\int_0^8 (x^2-1) dx$ using T_4

8. $\int_0^1 \sin(\pi x) dx$ using T_4

Identify which is larger without finding the estimates.

9. $\int_0^4 x^2 dx$ L_4 vs R_4

10. $\int_0^8 (x^2+1) dx$ T_4 vs L_4

11. $\int_0^{10} (-x+15) dx$ L_4 vs R_4

12. $\int_0^4 (-x^2+16) dx$ T_4 vs L_4

13. If $f'(x) > 0$ and $f(x) > 0$ for all real numbers then put the following in descending order: L_8, R_8, M_8

14. If $f''(x) > 0$ and $f(x) > 0$ for all real numbers, then T_4 is always an over or under estimate?

x	0	25	30	50
$f(x)$	4	6	8	12

20. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x) dx$ using three subintervals?

a. 290

b. 360

c. 380

d. 390

e. 430

21. Find the equivalent limit to $\int_5^{10} x^3 dx$.

22. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{5k}{n}\right)^4 \left(\frac{5}{n}\right) = \int_a^b f(x) dx$ What is $\int_a^b f(x) dx$?

23. Find the equivalent limit to $\int_2^8 x^2 dx$.

24. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(1 + \frac{4k}{n}\right) \left(\frac{4}{n}\right) = \int_a^b f(x) dx$ What is $\int_a^b f(x) dx$?

25. Find the equivalent limit to $\int_3^7 e^x dx$.

26. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(3 + \frac{4k}{n}\right) \left(\frac{4}{n}\right) = ?$

27. Find $\int_5^{11} \cos(x) dx$ in terms of an infinite limit, then expand the sigma expression for the first three terms.

28. For each of the three models given below (left, right and trapezoidal), write “over” if the model yields an over estimate and “under” if the model yields an under estimate.

f'	f''	Left Riemann	Right Riemann	Trapezoidal
+	+			
+	-			
-	+			
-	-			

Answers

1. 46

16. B

2. 1936

17. D

3. 536

18. C

4. 14.556

19. B

5. 13.410

20. A

6. 6

$$21. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(5 + \frac{5k}{n}\right)^3 \left(\frac{5}{n}\right)$$

7. 168

$$22. \int_2^7 x^4 dx$$

$$8. \frac{1 + \sqrt{2}}{4}$$

$$23. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{6k}{n}\right)^2 \left(\frac{6}{n}\right)$$

9. R_4 (increasing on this interval)

$$24. \int_1^5 \ln x dx$$

10. T_4 (concave up on this interval)

11. L_4 (decreasing on this interval)

$$25. \lim_{n \rightarrow \infty} \sum_{k=1}^n e^{\left(3 + \frac{4k}{n}\right)} \left(\frac{4}{n}\right)$$

12. L_4 (decreasing and concave down on this interval)

$$26. \int_3^7 \sin x^4 dx$$

13. $R_8 > M_8 > L_8$

14. T_4 is an over estimate

15. C

$$27. \lim_{n \rightarrow \infty} \sum_{k=1}^n \cos\left(5 + \frac{6k}{n}\right) \left(\frac{6}{n}\right) = \left(\frac{6}{n}\right) \left(\cos\left(5 + \frac{6}{n}\right) + \cos\left(5 + \frac{12}{n}\right) + \cos\left(5 + \frac{18}{n}\right) + \dots\right)$$

28. Table Below

f'	f''	Left Riemann	Right Riemann	Trapezoidal
+	+	Under	Over	Over
+	-	Under	Over	Under
-	+	Over	Under	Over
-	-	Over	Under	Under